

# Reputation and Accountability in Repeated Elections\*

Rainer Schwabe<sup>†</sup>

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## Abstract

This paper studies a model of infinitely repeated elections in which voters attempt simultaneously to select competent politicians and to provide them with incentives to exert costly effort. Voters are unable to incentivize effort if they base their reelection decisions only on incumbent reputation. However, equilibria in which voters use reputation-dependent performance cutoffs (RDC) to make reelection decisions exist and support positive effort. In these equilibria, politicians' effort is decreasing in reputation, and expected performance is decreasing in tenure. Like the equilibria in Ferejohn 1986, RDC equilibria rely on voters being indifferent between reelecting incumbents and electing challengers. I show that this voter-indifference condition is closely related to weak renegotiation-proofness (Farrell and Maskin 1989).

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<sup>†</sup>PhD candidate - Department of Economics, Princeton University. Email: rschwabe@princeton.edu.

## 1. Introduction

In a representative democracy, voters have the power to choose which citizens will occupy government posts. Even if they cannot directly observe politicians' actions, voters may harness this power to induce incumbent politicians to work in their interest by conditioning reelection on performance. This understanding of the relationship between voter and politician has been studied by Key (1966), Barro (1973), Ferejohn (1986), and others, and is the driving force behind all models of political agency.

If, as seems likely, politicians differ in their ability or preferences, an additional consideration must be taken into account by voters when making reelection decisions. There is a trade-off between having a reelection rule which effectively aligns the interests of the incumbent with the voters', and one which focuses on reelecting the type of politicians who are most able or willing to work in the voters' interest. The first of these is commonly referred to as sanctioning, while the second is called selection.

At an intuitive level, the two goals need not be entirely at odds. If good performance is the primary means by which voters can identify 'good' politicians, then focusing on selection means rewarding good performance with reelection. This should motivate all politicians to work in the voters' interest as 'bad' politicians try to appear 'good' in order to secure another term in office. Thus, selection and sanctioning are at least partly complementary.

In spite of this apparent complementarity, the view that elections are best understood in terms of selection only has gained considerable traction among political scientists<sup>1</sup>. To cite a representative and influential example, Fearon (1999, p. 77) writes: "when it comes time to vote it makes sense for the electorate to focus completely on the question of type: which candidate is more likely to be principled and share the public's preferences?" He argues that, while voters might

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<sup>1</sup>See the literature review below and Ashworth, Bueno de Mesquita, and Friedenberg (2009) for further discussion.

like to use a retrospective voting rule which incentivizes incumbents optimally, they cannot commit to doing so because politicians who are more likely to be ‘good’ are also more likely to perform well in the future. Thus, if voters are rational, they will focus exclusively on keeping ‘good’ politicians in office.

While this argument is unqualifiedly true in the model studied by Fearon, it is important to keep in mind the assumptions on which it relies. As I show in the literature review below, these assumptions are common in many related works. First, Fearon studies a two period model, so that electoral incentives cannot be provided during an incumbent’s second term. This assumption makes sense in many contexts, but it is not apt for studying settings where there are no term limits, such as the U.S. Congress. Second, he assumes that differences among politicians are such that some will perform better than others even in the absence of electoral incentives. This seems natural when studying differences in integrity or preferences, but it is not clear that it is true when politicians differ only in their competence or ability.

If one modifies these assumptions, it may no longer be true that ‘good’ politicians are always more likely to perform well in the future. Rather, future performance will depend on expectations of future behavior. Therefore, one can no longer conclude that voters must focus entirely on selection without looking closely at how voters and politicians expect their relationship to proceed. For example, if voters always reelect incumbents with high enough reputation, once a politician has developed a strong reputation for being ‘good’ he will have little reason to worry about his job security, and will thus have little motivation to exert costly effort. Therefore, voters may prefer to take their chances with an inexperienced politician who will work hard in order to make a name for herself rather than reelect a venerable incumbent who is not motivated to perform.

In this paper I study a simple, infinite-horizon model of repeated elections with no term limits in which politicians differ in their competence. I find that, in this setting, there is no equilibrium in which competent politicians exert positive effort while voters condition reelection only on reputation. Furthermore, I find a

class of equilibria in which voters use performance cutoffs to induce incumbents to work in their interest. These equilibria predict that politicians will work less as their reputation improves.

There are two types of politician in my model: H (high) and L (low). H-types are competent: by exerting costly effort they can improve the expected utility of voters. L-types, on the other hand, are incompetent: they do not have the ability to improve outcomes, or it is prohibitively costly for them to do so. If H-types are believed to exert some effort, the voters' beliefs about the likelihood of an incumbent being an H-type will evolve along with his observed performance. I refer to these beliefs as a politician's reputation.

Because of the repeated nature of the elections, the set of equilibria of this model is large and complex. In fact, any pure reelection strategy may be supported as part of a sequential equilibrium (see Proposition 1). This does not mean that any level of effort can be sustained in equilibrium. Nevertheless, the fact that arbitrary behavior on the part of voters can be derived as a prediction of this model highlights the importance of equilibrium selection. The incumbent's reputation is a payoff-relevant state variable in this model, so the Markov perfection refinement has some intuitive appeal. Furthermore, in a Markov perfect equilibrium, the voters' reelection decision depends only on the incumbent's reputation so that these equilibria can be interpreted as those in which voters focus only on selection. In the first of my main results (Proposition 2), I establish that the set of Markovian strategies is not rich enough to allow the voter to incentivize politicians to provide effort.

A slightly more permissive refinement, which has the added benefit of having a clear interpretation in a political context, is Weak Renegotiation-Proofness (WRP, Farrell and Maskin 1989). The logic behind it is as follows: if the relationship between an incumbent of a given reputation and voters can proceed in two different ways (e.g., reelect or not reelect the incumbent), it cannot be that the payoffs associated with one are strictly higher than the payoffs associated with the other for both the incumbent and the voter. If they were, incumbent and voter could

come to an agreement, i.e. renegotiate, to proceed in the mutually beneficial way. In particular, WRP rules out equilibria in which voters throw incumbents with good reputations out of office even if, under different circumstances, they would perform better in office than a challenger. Typically (e.g. Banks and Sundaram 1993), these equilibria rely on a belief that, if the incumbent is reelected, he will shirk, expecting to be thrown out of office regardless of his productivity.

To develop some intuition about what WRP might mean in a political context, consider the following example. Keep in mind that this example is meant to clarify the concept of WRP and is not meant to provide an explanation of actual events. Suppose voters in Washington D.C. had certain standards for behavior and outcomes which, if violated by an incumbent, would lead them to elect a relatively unknown challenger in the next election. Furthermore, assume that these standards of behavior included zero-tolerance for illegal acts. From 1979 to 1990, Marion Barry served as mayor and met these performance standards while developing a reputation for being a capable politician. However, in 1990 he was accused and convicted of drug use and possession as well as tax evasion, behavior which clearly violated our assumed standards of behavior for the voters. Nevertheless, in 1995, after working through his legal troubles, Barry was once again elected mayor of D.C.

One interpretation of this type of voter behavior is that Barry was able to convince voters that, if elected, he would behave as if he expected to be held to the same performance standard that he would have been held to had he not broken the law. Because he had a reputation for political ability, this meant that voters could expect a better performance from him, on average, than from an inexperienced challenger. In this sense, Barry and D.C. voters were able to renegotiate their implicit (and hypothetical) electoral contract. Clearly, if this type of renegotiation is feasible, any commitment by voters to expel high reputation politicians after they have violated performance standards is not credible if high reputation politicians are normally believed to outperform challengers.

WRP addresses a commitment problem quite similar to that highlighted by

Fearon (1999) and others and which I discuss above. If voters believe that there is a feasible and mutually beneficial way for their relationship with the incumbent to proceed, then it is not credible for the voters to commit to throw such an incumbent out of office. If it were the case that the best achievable future performance by an incumbent were increasing in reputation, rational voters would focus only on selection.

In this model, WRP is qualifiedly equivalent to the condition that the voters' expected payoffs be constant across incumbent reputations (see Claim 1 and Proposition 3 for details). If this is the case, voters face no commitment problem when making reelection decisions because they will be indifferent between having the incumbent or an inexperienced challenger in office. Note the similarity with the equilibria in a seminal work on political agency, Ferejohn 1986, in which voters commit to a reelection rule based on a fixed performance standard. In Ferejohn 1986 this voter indifference condition arises automatically from the assumption that politicians are identical. Thus, if one considers this assumption to be too strong, one may worry about the robustness of the proposed equilibria. However, I find that voter indifference has an important theoretical justification in a model with heterogeneous politicians. Thus, my results provide fresh perspective on, and microfoundations for, the equilibria of Ferejohn 1986.

My second main result (Theorem 1) establishes existence of a class of WRP equilibria in which H-types are incentivized to exert positive effort. In these equilibria, voters condition their reelection strategy only on reputation and current performance. Incumbents are reelected only if their observed performance exceeds a cutoff which varies with the incumbent's reputation at the beginning of his term. Crucially, these performance cutoffs vary in such a way as to make it incentive compatible for politicians to exert just enough effort to leave voters indifferent between reelecting the incumbent and electing an inexperienced challenger, thus making the voters' value function constant across reputations. I refer to this class of equilibria as equilibria in reputation-dependent performance cutoffs (RDC). I view RDC equilibria as a natural generalization of the strategies in Ferejohn 1986

because they rely on the same basic insights. First, performance cutoffs are an intuitive and effective way to provide incentives. Second, voters can credibly commit to using these strategies if they are indifferent between reelecting an incumbent and electing an inexperienced challenger.

An implication of voter indifference is that politicians are able to appropriate the benefits of increases in their reputation by exerting lower effort. This highlights a tension between the selection and incentivizing roles of elections. Voters could do better by committing to a reelection rule which optimally incentivized incumbents. However, such a commitment is not WRP and, thus, not credible. RDC strategies reconcile this tension in a way that is as simple as possible, while passing a stringent test of their credibility and providing politicians with incentives to exert costly effort.

Because, in a model with heterogeneous politicians, selection will play some role in explaining voter behavior, one may reasonably expect that a veteran politician who has developed a reputation for being of a certain type will be treated differently by voters than a first-termer with no record. This, in turn, suggests that a model of electoral control which simultaneously contemplates selection and sanctioning will help us understand the dynamics of political careers. That is, there is likely to be an interplay between an incumbent's reputation, tenure, and behavior, and the standards to which he is held by voters. In RDC equilibria, politicians of high reputation exert lower effort. Also, in expectation, reputation is positively related to tenure so that, for a given politician, tenure is negatively related to performance (see Claim 2).

The paper proceeds as follows. In the following Subsection, I discuss related work and its relationship to this paper. In Section 2, I describe the model and its assumptions. Section 3 addresses the problem of multiplicity of equilibria, and uses some simple equilibria of the model to motivate equilibrium selection criteria. In Section 4, I define RDC equilibria and prove their existence. In Section 5, I look at what RDC equilibria can tell us about the career dynamics of politicians. Section 6 concludes.

## 1.1. Related Literature

There is a growing number of papers which study the selection and incentivizing roles of elections in a unified framework. Much of this work builds on work by Holmström (1999) on career concerns, with the relationship most directly apparent in Persson and Tabellini (2000, ch. 4.5). Notable contributions include Reed (1994), Banks and Sundaram (1998), Fearon (1999), Berganza (2000), Ashworth (2005), and Besley (2006, ch. 3.3). Each of these works studies a model in which voters consider both the selection and incentivizing roles of elections and politicians face term limits. Additionally, several papers have applied similar models to the study of subjects such as transfers to special interest groups (Coate and Morris 1995 and Lohmann 1998), the incumbency advantage (Ashworth and Bueno de Mesquita 2006), constituency service (Ashworth and Bueno de Mesquita 2008), and CEO activism (Dominguez-Martinez, Swank, and Visser 2008) to name a few.

There are two important differences between the models cited in the previous paragraph and this paper. First, imposing term limits means that last period behavior is easily solved for, and reelection rules are derived by backward induction. In this paper there are no term limits, so voters and politicians face a dynamic problem at every stage. The second is the type of politician heterogeneity studied. In the papers above, voters are assumed to benefit from having a high type in office even if the prospect of reelection is not available to the voter as an incentivizing tool. In this paper, high types differ from low types only in their ability to induce outcomes preferred by the voters. However, improving outcomes is costly to high types so, in the absence of electoral incentives, average performance is equal for high and low types. I feel that this is a more natural way of modeling differences in ability or competence, while the alternative approach is best suited to modeling differences in honesty or alignment of preferences.

Banks and Sundaram (1993) study the selection and disciplining roles of elections in a fully dynamic framework with no term limits. However, they focus on stationary strategies in which voters hold all incumbents, regardless of reputation,

to a single performance standard. Therefore, there is no place in their analysis for career dynamics: for a given politician, effort and the probability of reelection are constant in tenure and reputation and independent of the history of play. Additionally, because it is supported by trigger-strategy punishments, the equilibria they propose are not weakly renegotiation-proof; they require that players follow continuation strategies whose associated payoffs are strictly Pareto dominated by other continuation strategies played in equilibrium. Snyder and Ting (2008) use a similar model to study how voter oversight can limit the influence of special interest groups. Gallego and Pitchik (2004) use a related model to explain the timing of the overthrow of dictators.

Duggan (2000) and Banks and Duggan (2006) study a model of repeated elections in which politicians differ in their spatial policy preferences. Voters use the incentive of reelection to induce politicians to temper their policy choices while in office. However, because there is no uncertainty in the execution of policy and strategies are stationary, there is no evolution of beliefs about the incumbent's preferences beyond their first period in office.

Meirowitz (2007) proposes a model of repeated elections in which two long-lived parties, differing in their policy preferences and valence, compete in elections each period. Voters are uncertain about the set of feasible policies rather than about the parties' characteristics or the policy choices made. He shows that, while electoral control is impossible if voters are constrained to using pure strategies, perfect control is possible in mixed strategies. If mixed strategies are to be used, each party must provide the same expected utility to the voter when in office. This leads to a voter indifference condition analogous to the one emphasized in this paper.

Smart and Sturm (2006) present a model of repeated elections in which incumbents' actions are publicly observable, but the underlying state of the world which determines which action is good for the voters is observed only by the incumbent. In this context, they prove that the best Markov perfect equilibrium in the absence of term limits involves all politicians taking the same action regardless of

the state of the world. They go on to argue that imposing term limits may help voters by decreasing the incentives for politicians to conform. Their result on the limits of Markov perfect equilibria are in the spirit of the first main result of this paper. The existence of RDC equilibria in my model suggests that allowing voters to condition on more information than Markov perfection allows is an alternative way to increase their expected payoffs which may dominate term limits.

Finally, this paper is related to the literature on dynamic principal-agent interactions outside of the political sphere. The approach taken here differs from that taken in much that literature in two main dimensions. First, this paper focuses on the use of a retention rule rather than a compensation contract as an incentivizing mechanism. Second, in most of the literature on principal-agent relationships the principal is assumed to be a Stackelberg first-mover, leaving the agent only his reservation utility. In this paper, I look at Nash equilibria which admit the possibility that the gains from interaction may be shared. Indeed, in the RDC equilibria which we focus on, the agent reaps all of the benefits from increases in his reputation and enjoys utility strictly greater than his reservation value.

## 2. The Model

I study a discrete-time, infinite horizon model of a democratic society. In order to focus on the problems of selecting competent politicians and providing them with incentives to perform well, I abstract from ideological differences in the electorate. Instead, I model citizens as a single, infinitely-lived representative voter.

### 2.1. Preferences, Timing, and Information in the Stage Game

Each period (indexed by  $t \in \{1, 2, \dots\}$ ), the voter must select a politician to carry out a task. There is an infinite set  $P$  of potential politicians from which the voter may choose. Each politician is infinitely-lived and may serve for as many periods, or terms, in office as the voter asks him to. Once replaced by a challenger, however, a politician cannot return to office.

After the voter elects a politician, the politician exerts effort  $a \in \mathbb{R}_+ = [0, \infty)$ . This effort impacts, but does not perfectly determine, results  $r \in \mathbb{R}$  which I interpret as the voter's stage-game utility.

In order to consider differences in competence, I assume that politicians are one of two types: H or L. For H-types, effort is related to results via a conditional distribution function  $F(r|a)$  with density  $f(r|a)$ . For ease of notation, I normalize units of effort so that effort exerted equals expected results:  $E(r|a) = \int_{-\infty}^{\infty} r f(r|a) dr = a$ .

H-types receive per-period utility  $u(a)$  when in office, and 0 otherwise. The utility function  $u(a)$  is twice continuously differentiable and strictly concave. Effort is costly so that  $u$  is weakly decreasing in  $a$  with  $u'(0) = 0$  and  $u'(a) < 0 \forall a > 0$ . I also assume that  $u(a) > 0$  for all  $a \in [0, \bar{a})$  and some  $\bar{a} > 0$ .

L-type politicians, on the other hand, are unable<sup>2</sup> to affect the distribution of  $r$  so that it is always  $F(r|0)$  when an L-type is in office. They receive a payoff  $u_L > 0$  when in office and 0 otherwise, so that they are always willing to serve if elected. Because L-types are always willing to hold office but cannot make choices which influence payoffs in this game, I will focus on the behavior of H-types.

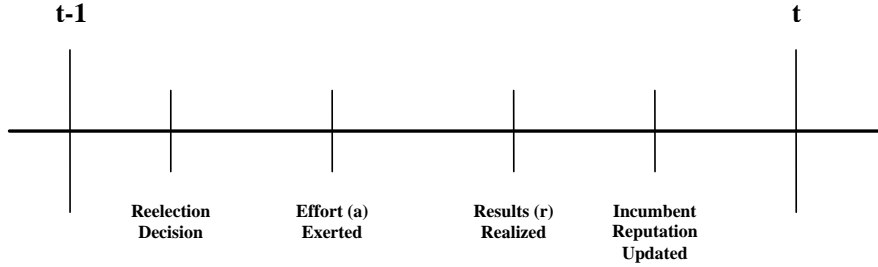
As is standard in games with imperfect monitoring (Abreu, Pearce, and Stachetti 1990), I assume that the distribution of results has full support:  $f(r|a) > 0$  for all  $r$  and  $a$ . This guarantees that effort levels can never be perfectly inferred by observing results. I also make the following assumptions for analytical convenience. First, that  $f(r|a)$  is twice continuously differentiable in both arguments. Second, that changing  $a$  does not change the shape of the distribution:  $f(r|a) = f(r + k|a + k)$  for any  $k \in \mathbb{R}$ . This also implies that outcomes can be written as the sum of the effort choice and a zero-mean stochastic component ( $\varepsilon$ ), a common modeling choice:  $r = a + \varepsilon$ . Finally, I assume that  $f(r|a)$  is symmetric around its mean.

Because the evolution of beliefs about incumbent types is central in this paper,

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<sup>2</sup>Alternatively, effort is too costly for L-types for it to be worthwhile exerting.  $-u'_L(0) > \frac{\delta}{1-\delta} u_L(0) f(0|0)$  is sufficient for this if I make the same assumptions on  $u_L$  as on  $u$ .

## The Stage Game



it is useful to make assumptions ensuring that good results are more likely when effort is high. Thus, I assume that  $f(r|a)$  satisfies the monotone likelihood ratio property (Milgrom 1981):  $\frac{f(x|a)}{f(x|a')} > \frac{f(y|a)}{f(y|a')}$  whenever  $x > y$  and  $a > a'$ .

In order to guarantee that the politician's objective function is concave so that I may work with first order conditions, I make the following joint assumption on  $u(a)$  and  $f(r|a)$ :  $-u''(a) > \max_Q \int f_{aa}(r|a)Q(r)dr$  where  $Q$  is any function  $Q : \mathbb{R} \rightarrow [u(0), \frac{u(0)}{1-\delta}]$  and  $f_{aa}(r|a)$  is the second derivative of  $f(r|a)$  with respect to  $a$ . If  $f(r|a)$  is the density of the normal distribution with mean  $a$  and variance 1,  $-u''(a) > 0.4839 \left(\frac{\delta}{1-\delta}u(0)\right)$  for all  $a$  is a sufficient condition.

A politician's type is the private information of the politician. The voter assigns a probability  $\mu_j$  of being an H-type to politician  $j$ . I call  $\mu_j$  politician  $j$ 's *reputation*. For ease of notation, when referring to the incumbent's reputation I drop the subscript  $j$ . Note that the expected stage-game payoff to the voter when an H-type incumbent exerts effort  $a$  is  $\mu a$ , so that reputation is payoff relevant.

The proportion of H-types among the set of potential politicians  $P$  is  $\mu_0$ . Because new politicians are selected randomly from this set,  $\mu_0$  will also be the reputation of any politician at the beginning of his first term.

## 2.2. Histories, Strategies, and the Repeated Game

At time  $t$ , the voter and all politicians will have information about who has been in office and what rewards the voter has received in all previous periods,  $1, 2, \dots, t-1$ . I call this information a  $t$ -history and label it  $h_t$ . Let  $H$  denote the set of all possible  $t$ -histories.

A *reelection strategy* is a measurable function  $\sigma : H \rightarrow [0, 1]$  denoting the probability with which the voter will reelect the incumbent, conditional on all currently available information.

Similarly, an *effort strategy* is a measurable function  $a : H \rightarrow \mathbb{R}_+$  denoting the effort which a given politician will exert conditional on being in office and on all currently available information.

A belief function  $\hat{\mu} : H \rightarrow [0, 1]^\infty$  is a measurable function specifying the probability with which the voter believes each politician in  $P$  to be an H-type. For any politician  $i$  who has never previously held office,  $\hat{\mu}_i = \mu_0$  regardless of the  $t$ -history  $h_t$ . In equilibrium, beliefs about a politician's type evolve according to Bayes' rule:

$$\hat{\mu}_j(h_{t+1}) = \frac{\hat{\mu}_j(h_t)f(r_{t+1}|a(h_t))}{\hat{\mu}_j(h_t)f(r_{t+1}|a(h_t)) + (1 - \hat{\mu}_j(h_t))f(r_{t+1}|0)}$$

Because the distribution  $f$  satisfies the monotone likelihood ratio property,  $\hat{\mu}_j(h_t)$  is strictly increasing in  $r_t$ . For ease of notation, in what follows I drop the subscript when referring to beliefs about the incumbent so that  $\hat{\mu}(h_t)$  denotes the probability that the incumbent at time  $t$  is an H-type.

It is important to note that different histories can lead to the same incumbent reputation. I can group these together to define a coarser partition of the set of all histories as follows: if  $\hat{\mu}(h^1) = \hat{\mu}(h^2)$  for  $h^1, h^2 \in H$  then  $h^1, h^2 \subset \hat{h} \in \hat{H}$ . Note that  $h^1$  and  $h^2$  need not be of the same length. I will refer to this as a Markovian partition of histories and I will use this definition of  $\hat{H}$  in the following section to define the Markov perfection refinement.

Given a strategy profile  $(\sigma, a)$  and beliefs  $\hat{\mu}$ , the voter can compute his expected

future payoffs at  $h_t$ . Keeping in mind that  $\sigma$ ,  $a$  and  $\hat{\mu}$  denote functions while  $\sigma(h_t)$ ,  $a(h_t)$  and  $\hat{\mu}(h_t)$  are particular values, I write  $V(\sigma, a, \hat{\mu}; h_t)$  for the voter's value function. Letting  $h_{t+1}(r)$  denote the  $t+1$ -history reached from  $h_t$  after a result  $r$  is observed, it may be defined recursively:

$$V(\sigma, a, \hat{\mu}; h_t) = \hat{\mu}(h_t)a(h_t) + \delta \int_{-\infty}^{\infty} V(\sigma, a, \hat{\mu}; h_{t+1}(r))f(r|a(h_t))dr$$

Where  $\delta \in (0, 1)$  is a discount factor common to the voter and all politicians. Note that I do not explicitly write the reelection probability  $\sigma(h_{t+1}(r))$  here. Instead,  $h_{t+1}(r)$  captures whether the incumbent is reelected or an inexperienced politician of reputation  $\mu_0$  is elected.

Similarly, I denote the value function of an incumbent H-type politician  $Q(\sigma, a, \hat{\mu}; h_t)$ . It may be defined recursively as:

$$Q(\sigma, a, \hat{\mu}; h_t) = u(a(h_t)) + \delta \int_{-\infty}^{\infty} \sigma(h_{t+1}(r))Q(\sigma, a, \hat{\mu}; h_{t+1}(r))f(r|a(h_t))dr$$

Note that I explicitly write the reelection probability  $\sigma(h_{t+1}(r))$  in the politician's value function to highlight that the reelection decision determines whether an incumbent will receive  $Q(\sigma, a, \hat{\mu}; h_{t+1}(r))$  the following period, or 0 if he is not reelected.

**Definition 1.** *A sequential equilibrium (Kreps and Wilson 1982) is a strategy profile  $(\sigma^*, a^*)$  and a belief function  $\hat{\mu}$  such that:*

1.  $V(\sigma^*, a^*, \hat{\mu}; h_t) \geq V(\sigma', a^*, \hat{\mu}; h_t)$  for all  $\sigma'$  and  $h_t$ .
2.  $Q(\sigma^*, a^*, \hat{\mu}; h_t) \geq Q(\sigma^*, a', \hat{\mu}; h_t)$  for all  $a'$  and  $h_t$ .
3.  $\hat{\mu}$  evolves according to Bayes' rule<sup>3</sup> using the strategies  $(\sigma^*, a^*)$ .

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<sup>3</sup>The full support assumption ensures that Bayes' rule is always applicable since all histories are reached with positive density.

Figure 2.1: Summary of Important Notation

$r$	Voter's stage-game utility.
$a$	Politician's effort.
$u(a)$	Politician's stage game utility.
$f(r a)$	pdf of $r$ given $a$ .
$V$	Voter's value function.
$Q$	Politician's value function.
$\sigma$	Voter's reelection strategy.
$\mu$	Incumbent's reputation.

### 3. Equilibrium Selection

As in any infinitely repeated game, I expect there to be a large set of sequential equilibria. In this section, I discuss the problem of the multiplicity of equilibria and some possibilities for narrowing my focus to those equilibria which are most appealing. I begin with the following result which starkly outlines the problem of multiplicity. Then, I proceed by describing several classes of equilibria of this model and using them to motivate several equilibrium refinements.

**Proposition 1.** *Any pure reelection strategy  $\sigma$  can be supported as part of a sequential equilibrium.*

To see that this is true, I first identify the equilibrium with the lowest payoffs for all players in the equilibrium set, which I call an equilibrium in *grim strategies*. Suppose H-type politicians always choose  $a = 0$ . Then, the voter is left indifferent among all politicians and may choose any reelection rule. In particular, it is a best response for him never to reelect a politician, regardless of his performance. This reelection strategy makes  $a = 0$  a best response.

Next, I note that this equilibrium may be used as part of other sequential equilibria as a credible punishment to the voter for not following a prescribed

reelection strategy. Because the voter's expected payoff can never be worse than 0, the following is an equilibrium for any pure reelection strategy  $\sigma$ : the voter plays  $\sigma$  on the equilibrium path while politicians play a best response to  $\sigma$ . If the voter ever deviates from  $\sigma$ , equilibrium play switches to grim strategies.

One may object to the equilibria above by arguing that it is implausible that all politicians in  $P$  will coordinate on playing grim strategies in the continuation game. Since the physical environment is identical each time a politician is elected to his first term, it seems natural to focus on equilibria in which strategies are the same every time the voter begins a fresh relationship with a politician. This, of course, implies that the value of the outside option for the politician is constant through all histories. In a sense, this is a stationarity condition which I will call *challenger-stationarity*. Because it is sufficient for my purposes and a weaker condition, I define challenger-stationarity in terms of the value of electing an inexperienced politician rather than the continuation strategies played.

**Definition 2.** *An equilibrium satisfies challenger-stationarity if the value of electing an inexperienced politician is history-independent.*

In a closely related paper, Banks and Sundaram (1993) describe an equilibrium of the repeated elections game which satisfies challenger-stationarity (following Banks and Sundaram, I call these *simple equilibria*). All politicians are held to a single performance standard. When this performance standard is not met, the politician is not reelected. This is the case for politicians of any reputation, even though the expected rewards to the voter are increasing in the incumbent's reputation. This is enforced through the following trigger strategy: after a politician has missed his performance target once, he never expects to be reelected again and will therefore never again exert effort.

A serious criticism of simple equilibria, in my view, is that after a politician with high reputation misses a performance target, both the voter and the politician would benefit from agreeing to keep the politician in office and continue play as if the incumbent had not violated the voter's performance standard. Therefore,

the punishment prescribed by the equilibrium is not credible. More formally, the equilibria are not *Weakly Renegotiation-Proof* (Farrell and Maskin 1989)<sup>4</sup>. There is a continuation equilibrium with associated payoffs which strictly Pareto dominate those specified as following the history in question.

Farrell and Maskin's definition of WRP equilibrium is as follows: an equilibrium strategy  $\sigma$  is WRP if there do not exist continuation equilibria  $\sigma^1$  and  $\sigma^2$  of  $\sigma$  such that  $\sigma^1$  strictly Pareto dominates  $\sigma^2$  (i.e. payoffs under  $\sigma^1$  are strictly greater for both players than under  $\sigma^2$ ). To adapt the definition of WRP to the current game, I must take into account that the politician's reputation is payoff relevant, so that continuation payoffs when the politician's reputation is  $\mu$  may not be feasible when his reputation is  $\mu' \neq \mu$ . The following definition formalizes this notion.

**Definition 3.** *A sequential equilibrium is Weakly Renegotiation-Proof (WRP) if, for any two histories  $h^1, h^2 \in H$  leading to a reputation  $\mu$ , i.e.  $h^1, h^2 \subset \hat{h} \in \hat{H}$ ,  $V(h^1) > V(h^2)$  implies  $Q(h^1) \leq Q(h^2)$  (and therefore  $Q(h^1) > Q(h^2)$  implies  $V(h^1) \leq V(h^2)$ ).*

As is well known, any Markov perfect equilibrium is WRP. Furthermore, given that reputation is the only payoff relevant state variable, it is natural to look for Markov perfect equilibria in which strategies depend only on reputation. In addition to the standard arguments for Markov perfect equilibrium (Maskin and Tirole 2001) which focus on the simplicity of Markovian strategies, it is also important for us to address this possibility because previous work on related models

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<sup>4</sup>Weak renegotiation-proofness is a condition of *internal consistency* in that it makes comparisons between the continuation payoffs of a given equilibrium strategy profile. Competing notions of renegotiation-proofness, such as that advocated by Pearce (1987), call for *external consistency* so that comparisons are made across equilibria. In particular, Pearce argues that comparisons should be made among the the lowest continuation payoffs of equilibria. Because not reelecting politicians (giving them continuation payoff of zero) is the voter's only effective tool for providing incentives, this approach is unlikely to narrow the set of equilibria in this game.

has tended to predict that voters will use a simple reputation cutoff as a reelection rule<sup>5</sup>. Additionally, related work on repeated elections by Meirowitz (2007), Duggan (2000), and Banks and Duggan (2006) has focused on Markov perfect equilibria. Banks and Sundaram '93 (p. 310) end their article by asking whether ‘interesting’ equilibria which are stationary in reputation exist. Proposition 2 below answers in the negative, at least for this slightly simpler setting.

**Definition 4.** *A sequential equilibrium is Markov perfect if strategies  $(\sigma, a)$  are measurable with respect to the Markovian partition  $\hat{H}: (\sigma, a) : \hat{H} \rightarrow [0, 1] \times \mathbb{R}_+$ .*

Markov perfection takes the idea that history can matter only through the state variable even further than WRP<sup>6</sup>. Once again, existence is easy to check as equilibrium in grim strategies provides a trivial example of a Markov perfect equilibrium. However, the following result makes clear that the Markovian criterion is too strict to allow for the voter to effectively incentivize H-type politicians.

**Proposition 2.** *There is no Markov perfect equilibrium with positive value for the voter ( $V > 0$ ).*

A full proof is provided in the Appendix (Section 7.2). To develop some of the intuition behind the proof, suppose that politicians of all reputations provide effort of at least  $\alpha > 0$  in equilibrium. If reelection strategies depend only on reputation, the politician’s ex-ante value of acquiring a reputation  $\mu$  is  $\sigma(\mu)Q(\mu) = \hat{Q}(\mu)$ . Because posterior reputation is increasing in performance, in order to provide incentives for effort the function  $\hat{Q}(\mu)$  must be increasing in reputation. As a politician’s reputation nears 1, the change in his reputation for a fixed but wide

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<sup>5</sup>See Reed (1994), Banks and Sundaram (1998), Fearon (1999), Berganza (2000), and Ashworth (2005). In these models with term limits, it is assumed that high types perform better than low types in the last term in which no incentives for effort can be provided. Therefore, incumbents are reelected if their expected type is higher than that of a replacement.

<sup>6</sup>See Farrell and Maskin 1989 for further discussion of the relation between WRP and Markov Equilibria.

set of outcomes ( $r$ ) approaches 0. Therefore, the politician's value function  $\hat{Q}$  must increase at least a fixed amount (itself dependent on  $\alpha$ ) in each of an infinity of ever smaller intervals. However, I know that  $\hat{Q}$  is bounded above by the value of holding office forever while exerting zero effort:  $\frac{u(0)}{1-\delta}$ . Therefore, providing incentives for effort at least  $\alpha$  for all reputations is infeasible. Conversely, if politicians of reputation at least  $\mu$  do not provide effort, it is not worthwhile for the voter to reelect them. This in turn, means that politicians should avoid ending up with a reputation higher than  $\mu$ , and they can only do this by providing lower effort, leading to an unraveling of incentives for incumbents of all reputations.

If one persists in looking for equilibria in which positive effort is exerted while insisting that strategies depend on history in the simplest way possible, the natural next step is to allow for strategies to depend on both the politician's current reputation and his reputation the previous period. Such strategies are Markovian if we take  $(\mu_{t-1}, \mu_t)$  rather than  $\mu_t$  as the state variable. This condition is equivalent to constraining strategies to depend only on reputation and performance:  $(r_t, \mu_t)$  or  $(\mu_{t-1}, r_t)$ . In Section 4, I define a class of equilibria which satisfy this condition, discuss their relation to previous work, and prove their existence. These equilibria are also WRP.

#### **4. Equilibria in Reputation-Dependent Performance Cutoffs (RDC)**

Because the strategies I will consider in this section depend only on reputation at the beginning of the term and current performance, I drop the notation emphasizing the dependence of the voter's and the politicians' value functions  $V$  and  $Q$  on the entire history of play and strategy profiles. Instead, I emphasize their dependence on incumbent reputation by writing  $V(\mu)$  and  $Q(\mu)$ .

In order to find equilibria in which the voter provides incentives for H-types to provide positive effort but that are WRP and depend on history in the simplest way possible, I look to the structure of the equilibria in the baseline models of po-

litical agency. In my view, this has the added virtue of providing some continuity in the modeling and understanding of electoral incentives. The seminal work of Ferejohn 1986 makes two important observations:

- Performance cutoffs are effective means of providing incentives to politicians.
- Voter indifference over incumbents and replacements can be exploited to sustain equilibria with performance cutoffs.

In this model, politicians differ only in their perceived probability of being an H-type - their reputation. An incumbent's reputation will evolve as his record of performance grows and, once he has served at least one term, it will never (with probability zero) be exactly the same as that of a challenger ( $\mu_0$ ). Therefore, for the voter to be kept indifferent between reelecting an incumbent and electing a challenger, it must be that politicians of different reputations provide the same expected utility to the voter:  $V(\mu) = V(\mu_0)$ , at least for  $\mu > \mu_0$ . Therefore, I speak of voter indifference and a constant voter value function interchangeably.

Intuitively, this voter indifference condition can be seen as a formalization of the often-voiced sentiment: "One politician is as bad as another." This does not mean that there are no differences in competence among politicians, but that they all exploit the system in their favor to the point where expected performance is constant across politicians.

In addition to the connection to earlier models of political accountability, the voter indifference condition is connected in the current model to the concept weak renegotiation-proofness (WRP, Definition 3). Clearly, voter indifference implies WRP since continuation payoffs are the same for the voter after any history of play, ruling out Pareto improvements.

**Claim 1.** *Any equilibrium in which  $V(\mu) = V(\mu_0)$  for all  $\mu$  which are reelected with positive probability is weakly renegotiation-proof (WRP).*

The following Proposition goes some way toward establishing the reverse implication; i.e. that WRP implies voter indifference. Specifically, the indifference

condition will hold for a set of reputations of positive measure, and strategies outside of this set will be "uninteresting". In order to do so, I assume that the effort strategies of newly elected politicians do not depend on prior history (i.e. equilibria are challenger-stationary, see Definition 2). This seems natural in the current context where each time a politician is elected for the first time, the continuation game looks identical to the start of the game at time 0.

**Proposition 3.** *In any equilibrium satisfying weak renegotiation-proofness (WRP) and challenger-stationarity the following conditions hold:*

- *There is a subset of reputation space of strictly positive measure  $S \subset [0, 1]$  such that, for any  $\mu \in S$ , if  $\hat{\mu}(h) = \mu$  then  $V(h) = V(\mu_0)$ .*
- *For any  $\mu \in S^C = [0, 1] \setminus S$ , if  $h^1, h^2 \subset \hat{h}(\mu)$  then, either  $\sigma(h^1) = \sigma(h^2) = 1$  or  $\sigma(h^1) = \sigma(h^2) = 0$ . That is, strategies in the complement of  $S$  are Markovian and degenerate.*

**Proof.** If an equilibrium does not provide positive value for the voter, then the voter's value function is constant at 0 and the conditions above are trivially satisfied. Thus, in what follows, I look at equilibria in which  $V(\mu_0) > 0$ .

Consider any reputation  $\mu$  such that one can find histories  $h^1$  and  $h^2$  satisfying  $\hat{\mu}(h^1) = \hat{\mu}(h^2) = \mu$ ,  $\sigma(h^1) = 1$  and  $\sigma(h^2) = 0$  (or strategies are mixed but may lead to reelection after  $h^1$  and dismissal after  $h^2$ ). Then WRP implies that, because  $Q(h^1) > Q(h^2)$ ,  $V(h^1) \leq V(h^2)$ . Also, because it is a best response to reelection after  $h^1$ ,  $V(h^1) \geq V(\mu_0)$ . Because it is a best response not to reelection after  $h^2$ ,  $V(h^2) = V(\mu_0)$ . From this I conclude that  $V(h^1) = V(h^2) = V(\mu_0)$ .

This leaves reputation levels at which incumbents are always reelected or always thrown out of office. However, any reelection strategy leading to this sort of behavior over almost all reputations is an essentially Markovian reelection strategy. By the generalization of Proposition 2 in the appendix, this contradicts the premise that the equilibrium in question provides positive value for the voter. ■

As it relates to the model of Ferejohn 1986, the relationship between WRP and voter indifference solidifies the microfoundations of equilibria in performance cut-offs. Even if one allows for heterogeneity among politicians, there is an intuitively appealing equilibrium refinement (WRP) which leads back to voter indifference. Thus, its use as a commitment device is both credible and focal.

If we are to preserve voter indifference, we must use performance cutoffs which adjust to the incumbent's reputation. Otherwise, expected results will be increasing in reputation as in Banks and Sundaram 1993's simple equilibria.

In order to keep the voter indifferent between incumbents and replacements ( $V(\mu) = V(\mu_0)$ ), it must be that

$$V(\mu) = \mu a(\mu) + \delta \int_{-\infty}^{\infty} [\sigma(\hat{\mu}(r, \mu))V(\hat{\mu}(r, \mu)) + (1 - \sigma(\hat{\mu}(r, \mu)))V(\mu_0)] f(r|a(\mu)) dr$$

Solving for  $a(\mu)$  and substituting  $V(\mu) = V(\mu_0) = V$ , we find that  $V = \mu a(\mu) + \delta V$ . Solving for the incumbent's effort strategy:  $a(\mu) = \frac{V(1-\delta)}{\mu}$ . Denoting  $v = V(1 - \delta)$  I write the identity for effort levels which keep the voter indifferent among politicians as:

$$a(\mu) = \frac{v}{\mu}$$

I refer to  $v$  as the *value to the voter* of an effort profile  $a(\mu)$ . Note that  $a'(\mu) = -\frac{v}{\mu^2}$  so that effort is decreasing in reputation. Clearly, any equilibrium with positive value to the voter ( $v > 0$ ) will involve a lowest reputation politician which will ever be elected, since  $a(\mu) \rightarrow \infty$  as  $\mu \rightarrow 0$ . I denote this lowest reelectable reputation  $\mu_{\min}$ .

Because effort strategies  $a(\mu)$  keep the voter indifferent among reelection strategies, if there exists a performance cutoff function  $r(\mu) : [0, 1] \rightarrow \mathbb{R}$  which makes  $a(\mu)$  a best response, this will be a sequential equilibrium.

**Definition 5.** *An equilibrium in reputation-dependent performance cutoffs (RDC) with value  $v$  is a sequential equilibrium in which:*

- *Politicians follow an effort strategy  $a(\mu_t) = \frac{v}{\mu_t}$ .*

- The voter follows a reelection strategy  $\sigma(r_t, \mu_t) = \begin{cases} 1 & \text{if } r_t \geq r(\mu_{t-1}) \\ 0 & \text{otherwise} \end{cases}$

The following Theorem states the existence of equilibria in reputation-dependent cutoff strategies.

**Theorem 1.** *There exists a class of equilibria in reputation-dependent performance cutoffs (RDC) in which the voters use a reputation-dependent performance cutoff as their reelection strategy, are indifferent among politicians of all reputations above some threshold  $\mu_{\min}$ , and receive strictly positive expected utility.*

The proof (in Section 7.1) proceeds as follows: let  $Q(\mu)$  be any bounded and well-behaved candidate for the politician's value function. If I have chosen  $v$  carefully, it will be obtainable under  $Q(\mu)$  in an RDC equilibrium since  $Q(\mu)$  is bounded below by  $u(0)$ . I then define an operator  $T(Q)(\mu) = u(a(\mu)) + \delta \int_{r_Q(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f(r|a(\mu)) dr$  where  $r_Q(\mu)$  is a reputation-dependent performance cutoff implementing  $v$ . A fixed point of  $T$  will be a value function  $Q$  with associated cutoff function  $r_Q(\mu)$  implementing an effort strategy  $a(\mu) = \frac{v}{\mu}$ . Because this effort strategy leaves the voter indifferent between reelecting the incumbent or not, the cutoff function describes a reelection strategy which is a best response. Therefore, once I check sufficient conditions for a fixed point of  $T$ , I have found an RDC equilibrium.

The equilibria constructed in the proof of Theorem 1 use performance cutoffs which are above the expected performance of high types (i.e.  $r(\mu) - a(\mu) > 0$ ), and therefore politicians are always reelected with probability strictly less than  $\frac{1}{2}$ . Because of the relatively low reelection probability, the politician's value function is lower than it would be in an equilibrium with higher reelection rates, and therefore the highest level of implementable effort would likely be higher in this alternative scenario. Generally, I would expect similar equilibria using cutoffs below expected performance to exist and guarantee reelection rates strictly higher than  $\frac{1}{2}$ . However, moving performance cutoffs below expected performance allows

for the possibility that you may be reelected when your reputation has decreased, and thus that a politician will be reelected even if it is infeasible for him to be incentivized to provide the required effort to keep indifference. Whether this takes place will depend on the slope of  $r(\mu)$ , which in turn depends on the shape of  $Q(\mu)$ , which is an endogenous object. Therefore, whether RDC equilibria with performance cutoffs below expected performance exist remains an open question.

## 5. Career Dynamics and Comparative Statics

A straight-forward implication of RDC equilibria is that effort decreases with reputation. Additionally, in any RDC equilibrium expected reputation increases with tenure. This is easy to see by the following argument. Because, in equilibrium, the voter correctly anticipates the incumbent's behavior as a function of his type, the expected reputation of the incumbent after serving a term is the same as his reputation at the beginning of the term. However, those incumbents who end the term with the lowest reputation will be thrown out of office, leading us to conclude that expected reputation will increase every time an incumbent is reelected. This implies a negative relationship between expected performance and tenure for a given politician (though not across politicians).

**Claim 2.** *For a given politician, expected performance is negatively related to tenure.*

This is a prediction which has been emphasized by others, including Banks and Sundaram (1998) and Ashworth (2005), though their derivation relies on last-period effects. As Ashworth (2005) points out, the prediction fits well with the negative correlation between tenure and personal constituent services in the U.S. House of Representatives examined in Cain, Ferejohn and Fiorina (1990). In a study of the U.S. senate, Levitt (1996) finds some evidence of a positive correlation between ideological shirking and tenure.

Recent work by Galasso, et al. (2009) finds a negative relationship between tenure and attendance in the Italian legislature. Attendance may be interpreted as an observation of performance in this context if I reinterpret the model to fit Italy's parliamentary system. In this case, politicians are directly accountable to their party rather than to the voters. One might imagine that parties face a similar retention problem to that faced by voters in democracies with direct representation, and thus may use RDC strategies.

Because of the importance of the voter's outside option in RDC equilibria, it should not be surprising that the best payoff achievable for voters in an RDC equilibrium is increasing in the average reputation of new politicians ( $\mu_0$ ). Indeed, given an RDC equilibrium under  $\mu_0$ , the same strategies may be used when new politicians are more likely to be H-types ( $\mu'_0 > \mu_0$ ). This is because RDC strategies do not depend on the initial reputation of politicians. This implies that the highest achievable voter utility is weakly increasing in  $\mu_0$ .

**Claim 3.** *The highest expected payoff to the voter in an RDC equilibrium is weakly increasing in the proportion of high types in  $P$  ( $\mu_0$ ).*

## 6. Conclusions

The aim of this paper has been to improve the general understanding of the dual role of elections: selecting competent politicians and incentivizing them to exert costly effort to the benefit of the electorate. In particular, I have focused on the potential interaction between a politician's reputation, the voter's willingness to replace him with a less experienced candidate, and the politician's performance. I have done so in the context of a simple model of repeated elections without term limits which does not assume that competence is desirable to the voter even in the absence of incentivizing mechanisms.

As in many infinitely repeated games, the problem of equilibrium selection takes center stage. However, attention paid to this issue has been rewarded in

unexpected ways. I have shed light on the question of whether voters can effectively incentivize politicians by simply conditioning reelection on reputation. The answer is no (Proposition 2), at least in the simple model I study. I have uncovered an interesting relationship between weak renegotiation-proofness and the condition that the voter be left indifferent among politicians of different reputations and, therefore, between reelecting an incumbent and electing an inexperienced challenger (Claim 1 and Proposition 3). This has given us fresh perspective on a seminal work in political agency (Ferejohn 1986) and increased confidence in its underlying logic. Finally, I have considered some of the virtues and limitations of the large set of equilibria in trigger strategies.

My exploration of the equilibrium set and its refinements led me to generalize the equilibria of Ferejohn 1986 to a model with non-homogeneous politicians (RDC equilibria, Section 4). The use of voter indifference to support performance cutoffs which, in turn, allow the voter to incentivize effort from politicians is consistent with several intuitively appealing equilibrium refinements. Additionally, after establishing existence (Theorem 1), I go on to explore the predictions of the model for political careers. The results presented in Section 5 replicate those derived in similar models with term limits and in which incumbent type directly affects voter utility. That they continue to hold when there are no term limits and politicians differ only in competence should encourage researchers to look for evidence of these career dynamics in contexts such as the U.S. Congress and understand them as a consequence of political agency.

I conclude by pointing to two promising avenues for future research. First, one might expect related models to yield rich predictions about the variation in reelection rates across politicians of differing tenure and reputation. Because of technical difficulties which I discuss at the end of Section 4, I have not been able to derive such implications from this paper's model. Second, a model of repeated elections which allows for differences in politicians' integrity instead of, or in addition to, differences in competence will make different predictions about voter behavior and political careers. In particular, I conjecture that, in stark contrast to

the results of this paper, stationary or Markovian strategies would serve the voter well if ‘good’ politicians perform as well as any incumbent can. Once a politician’s reputation is high enough, his expected performance will necessarily be better than that of a challenger, and voters would like to keep such an incumbent in office as long as possible.

## 7. Appendix

For easy reference in the proofs that follow, I rewrite and label the assumptions on the density function  $f(r|a)$  discussed in Section 2.1.

$$\text{Full support: } f(r|a) > 0 \text{ for all } r \text{ and } a. \quad (\text{A1.})$$

$$f(r|a) \text{ is twice continuously differentiable in both arguments.} \quad (\text{A2.})$$

$$\text{Monotone Likelihood Ratio Property (MLRP): } \frac{f(x|a)}{f(x|a')} > \frac{f(y|a)}{f(y|a')} \text{ whenever } x > y \text{ and } a > a'. \quad (\text{A3.})$$

$$\text{Immutability: } f(r|a) = f(r+k|a+k) \text{ for any } k \in \mathbb{R}. \quad (\text{A4.})$$

$$\text{Symmetry: } f(r|a) \text{ is symmetric around its mean.} \quad (\text{A5.})$$

$$\text{Strict Concavity: } -u''(a) > \max_Q \int f_{aa}(r|a)Q(r)dr \text{ for } Q : \mathbb{R} \rightarrow [u(0), \frac{u(0)}{1-\delta}]. \quad (\text{A6.})$$

It is useful to note that A3. and A4. imply that  $f(r|a)$  is log-concave in  $r$  (see Bagnoli and Bergstrom 2005 for some implications). I use this fact in the proof of Lemma 2 below.

**Lemma 1.** *If  $f(r|a)$  is twice continuously differentiable and it satisfies the monotone likelihood ratio property and immutability, it is log-concave in  $r$ .*

**Proof.** Let  $x' > x$  and  $y' > y$ .

A density function satisfies the monotone likelihood ratio property if:

$$\frac{f(x',y')}{f(x',y)} > \frac{f(x,y')}{f(x,y)}$$

Taking logs on both sides:

$$\ln f(x'|y') - \ln f(x'|y) > \ln f(x|y') - \ln f(x|y)$$

$$\text{If } f \text{ is twice continuously differentiable, } \ln f(x'|y') - \ln f(x'|y) \approx \frac{\partial \ln f(x|y)}{\partial y} (y' - y)$$

when  $(y' - y)$  is small. Thus, the inequality above implies:

$$\frac{\partial \ln f(x'|y)}{\partial y} > \frac{\partial \ln f(x|y)}{\partial y}$$

Because this must hold for all  $x' > x$ , this is equivalent to  $\frac{\partial^2 \ln f(x|y)}{\partial y \partial x} > 0$ .

Immutability states that  $f(x|y) = f(x - y|0)$ .

Therefore, it must be that  $\frac{\partial^2 \ln f(x|y)}{\partial y \partial x} = -\frac{\partial^2 \ln f(x-y|0)}{\partial x^2} > 0$ . Since this holds for all  $x$  and  $y$ , I conclude that  $f$  is log concave. ■

In what follows,  $f_a(r|a) = \frac{\partial f(r|a)}{\partial a}$ ,  $f_{aa}(r|a) = \frac{\partial^2 f(r|a)}{\partial a^2}$  and  $\hat{\mu}_2(r, \mu) = \frac{\partial \hat{\mu}(r, \mu)}{\partial \mu}$ .

### 7.1. Existence of RDC equilibria - proof of Theorem 1

I proceed by determining reputation-dependent performance cutoffs which implement effort levels which make the voter's expected utility constant across reputations. Once I have done this, I define an operator which, for any well-behaved candidate value function for the incumbent, determines performance cutoffs and a new candidate value function. A fixed point of this operator gives us an incumbent value function and associated performance cutoffs. Because, at every reputation point, the voter is indifferent between reelecting the incumbent and electing a challenger, using these performance cutoffs as a reelection strategy is sequentially rational for the voter. Thus, the following four elements describe a sequential equilibrium: value functions for the politician and the voter, effort strategies which keep the voter's value function constant, and reelection strategies which use the derived performance cutoffs to make reelection decisions. In order to guarantee the existence of a fixed point, I must check that the conditions for Schauder's fixed point theorem hold. I do so in a series of Lemmas.

When facing a reputation-dependent performance cutoff, an H-type politician with reputation  $\mu$  solves the problem:

$$\max_a \left\{ u(a) + \delta \int_{r(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f(r|a) dr \right\}$$

To implement performance  $v$  (or effort strategy  $a(\mu) = \frac{v}{\mu}$ ) with a reputation-dependent cutoff  $r(\mu)$  the politician's first order condition (FOC) must be satisfied at  $a(\mu)$ :

$$u'(a(\mu)) + \delta \int_{r(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f_a(r|a(\mu)) dr = 0$$

The FOC uniquely determines the incumbent's action since, by assumption A6.,

$$u''(a(\mu)) + \delta \int_{r(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f_{aa}(r|a(\mu)) dr < 0$$

The FOC must hold at every reputation point  $\mu$  so that the derivative of the F.O.C. with respect to  $\mu$  must be 0:

$$u''(a(\mu))a'(\mu) - \delta r'(\mu) Q(\hat{\mu}(r(\mu), \mu)) f_a(r(\mu)|a(\mu)) + \delta \int_{r(\mu)}^{\infty} Q'(\hat{\mu}(r, \mu)) \hat{\mu}_2(r, \mu) f_a(r|a(\mu)) + Q(\hat{\mu}(r, \mu)) f_{aa}(r|a(\mu)) a'(\mu) dr = 0$$

Solving for  $r'(\mu)$ :

$$r'(\mu) = \frac{\frac{1}{\delta} u''(a(\mu))a'(\mu) + \int_{r(\mu)}^{\infty} Q'(\hat{\mu}(r, \mu)) \hat{\mu}_2(r, \mu) f_a(r|a(\mu)) + Q(\hat{\mu}(r, \mu)) f_{aa}(r|a(\mu)) a'(\mu) dr}{f_a(r(\mu)|a(\mu)) Q(\hat{\mu}(r(\mu), \mu))} \quad (7.1)$$

The Fundamental Theorem of Differential Equations guarantees the existence of a function  $r(\mu)$  satisfying the equation above as long as the first order condition is feasible and I can bound  $r(\mu)$  away from the point where  $f_a(r(\mu)|a(\mu)) = 0$  (for symmetric distributions, this point is  $a(\mu)$ ), since the RHS of the expression above is continuous and the domain of  $r(\mu)$  is compact.

Before presenting a proof of existence of these equilibria, I select a feasible value for the voter:  $v > 0$ . For analytical convenience, I focus on cutoffs where

$r(\mu) - a(\mu) > 0$  and  $f_a(r(\mu)|a(\mu)) > 0$ .

A lower bound for the value of holding office is  $\bar{Q} = u(0)$ . To emphasize its dependence on  $v$ , I write  $a(\mu, v) = \frac{v}{\mu}$  for the incumbent's effort strategy. Using this lower bound as a hypothetical constant value function and invoking the immutability assumption A4.:

$$-u'(a(\mu, v)) = \delta \int_{r(\mu)}^{\infty} \bar{Q} f_a(r|a(\mu, v)) dr = \delta \bar{Q} f(r(\mu)|a(\mu, v))$$

Clearly, this equality cannot hold for  $v$  large enough. However, as  $v \rightarrow 0$ ,  $a(\mu, v) \rightarrow 0$  and therefore  $u'(a(\mu, v)) \rightarrow 0$ . However,  $\bar{Q} > 0$ , so that the equation must hold for appropriate  $r(\mu)$  for  $v$  low enough (but still strictly positive). Indeed, I can guarantee that a strictly positive  $v$  may be sustained as above even if I restrict attention to cutoffs satisfying  $r(\mu) - a(\mu) > L$  for any given lower bound  $L$ . This will be useful when proving Lemma 2.

I now present the fixed point problem, referring to the derivations above as they become useful.

**Definition 6.** Let  $C([0, 1])$  be the space of bounded, continuous functions  $f : [0, 1] \rightarrow \mathbb{R}$ .

Let  $\hat{C} \subset C([0, 1])$  be the restriction of this space to functions with  $K$ -bounded first derivative and codomain  $[u(0), \frac{u(0)}{1-\delta}]$ .

It is clear that  $\hat{C}$  is non-empty, bounded, closed, and convex.

**Definition 7.** The operator  $T : \hat{C} \rightarrow \hat{C}$  is:

$$T(Q)(\mu) = u(a(\mu)) + \delta \int_{r_Q(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f(r|a(\mu)) dr$$

A fixed point of this operator will define a value function for the politician in a reputation-dependent cutoff equilibrium. To prove the existence of a fixed point, I will use Schauder's fixed point theorem. Schauder's theorem is a generalization of Brouwer's fixed point theorem to infinite-dimensional spaces. For a proof, see Lusternik and Sobolev (1974).

**Theorem 1 (Schauder's Fixed Point Theorem).** *Let  $X$  be a bounded subset of  $\mathbb{R}^m$ , and let  $C(X)$  be the space of bounded continuous functions on  $X$ , with the sup norm. Let  $F$  be nonempty, closed, bounded and convex. If the mapping  $T : F \rightarrow F$  is continuous and the family  $T(F)$  is equicontinuous, then  $T$  has a fixed point in  $F$ .*

I must first verify that  $T$  maps  $\hat{C}$  to  $\hat{C}$ .

That  $T(Q)$  is continuously differentiable in  $\mu$  is immediate from the differentiability of  $f$ ,  $Q$ ,  $a(\mu)$ , and  $r_Q(\mu)$ .

That  $T(Q)$  has a  $K$ -bounded derivative is verified in the following Lemma.

It will be useful in proving the Lemma to note that the first derivative with respect to  $\mu$  of the Bayesian updating function is:

$$\frac{\partial \hat{\mu}(r, \mu)}{\partial \mu} = \hat{\mu}_2(r, \mu) = \frac{f(r|a(\mu))f(r|0) + \mu(1 - \mu)a'(\mu)f_a(r|a(\mu))f(r|0)}{(\mu f(r|a(\mu)) + (1 - \mu)f(r|0))^2}$$

It is useful to note that  $\hat{\mu}_2(r, \mu) \rightarrow 1$  as  $a(\mu) \rightarrow 0$ . The second term in the numerator converges to zero since  $f_a(r|a(\mu))f(r|0)$  is uniformly bounded above.

**Lemma 2.** *For any continuously differentiable function  $Q$  with absolutely  $K$ -bounded first derivative,  $\left| \frac{\partial T(Q)}{\partial \mu} \right| < K$  for any  $\mu \in [\mu_0, 1]$  and small enough  $v > 0$ .*

**Proof.**  $\frac{\partial T(Q)}{\partial \mu} = \frac{\partial [u(a(\mu)) + \delta \int_{r(\mu)}^{\infty} Q(\hat{\mu}(r, \mu))f(r|a(\mu))dr]}{\partial \mu} =$

$$u'(a(\mu))a'(\mu) + \delta \int_{r(\mu)}^{\infty} a'(\mu)Q(\hat{\mu}(r, \mu))f_a(r|a(\mu))dr + \delta \int_{r(\mu)}^{\infty} Q'(\hat{\mu}(r, \mu))\hat{\mu}_2(r, \mu)f(r|a(\mu))dr - \delta r'(\mu)Q(\hat{\mu}(r(\mu), \mu))f(r(\mu)|a(\mu))$$

The first two terms add up to zero by the politician's F.O.C. Substituting equation 7.1 into the fourth term:

$$\delta \int_{r(\mu)}^{\infty} Q'(\hat{\mu}(r, \mu))\hat{\mu}_2(r, \mu)f(r|a(\mu))dr - \frac{f(r(\mu)|a(\mu))}{f_a(r(\mu)|a(\mu))} \left( u''(a(\mu))a'(\mu) + \delta \int_{r(\mu)}^{\infty} Q'(\hat{\mu}(r, \mu))\hat{\mu}_2(r, \mu)f_a(r|a(\mu)) + Q(\hat{\mu}(r, \mu))f_{aa}(r|a(\mu))a'(\mu)dr \right)$$

I first consider the terms which include  $Q'$ . Combining them gives:

$$\begin{aligned}
& \left| \int_{r(\mu)}^{\infty} \hat{\mu}_2(r, \mu) Q'(\hat{\mu}(r, \mu)) \left( f(r|a(\mu)) - \frac{f(r(\mu)|a(\mu))}{f_a(r(\mu)|a(\mu))} f_a(r|a(\mu)) \right) dr \right| \\
& < K \left| \int_{r(\mu)}^{\infty} \hat{\mu}_2(r, \mu) \left( f(r|a(\mu)) - \frac{f(r(\mu)|a(\mu))}{f_a(r(\mu)|a(\mu))} f_a(r|a(\mu)) \right) dr \right| \\
& < K \left| \frac{f(r(\mu)|a(\mu))}{f_a(r(\mu)|a(\mu))} \int_{r(\mu)}^{\infty} \hat{\mu}_2(r, \mu) f_a(r|a(\mu)) dr \right|
\end{aligned}$$

Where I use Lemma 1 to derive both inequalities as it guarantees that the terms involving  $f(r|a(\mu))$  and  $f_a(r|a(\mu))$  will not change sign.

Because  $\lim_{a \rightarrow 0} \hat{\mu}_2(r, \mu) = 1$  and using Lemma 1 again I can say that the last expression is finite for any  $r(\mu) > a(\mu)$  and low enough  $a(\mu)$ . Therefore,  $\lim_{r(\mu) \rightarrow \infty} \frac{f(r(\mu)|a(\mu))}{f_a(r(\mu)|a(\mu))} \int_{r(\mu)}^{\infty} \hat{\mu}_2(r, \mu) f_a(r|a(\mu)) dr = 0$ .

As argued in the text preceding the Lemma, since  $Q$  is bounded below, I may choose  $r(\mu)$  as large as I like while still supporting positive effort. In particular, I may choose  $r(\mu)$ , and thus  $a(\mu)$ , so that the following inequality holds for all  $\mu > \mu_0$ :

$$\delta \int_{r(\mu)}^{\infty} \hat{\mu}_2(r, \mu) f(r|a(\mu)) dr - \frac{f(r(\mu)|a(\mu))}{f_a(r(\mu)|a(\mu))} \delta \int_{r(\mu)}^{\infty} \hat{\mu}_2(r, \mu) f_a(r|a(\mu)) dr < .9$$

Therefore,  $\delta \int_{r(\mu)}^{\infty} \left[ Q'(\hat{\mu}(r, \mu)) f(r|a(\mu)) - Q'(\hat{\mu}(r, \mu)) \frac{f(r(\mu)|a(\mu))}{f_a(r(\mu)|a(\mu))} f_a(r|a(\mu)) \right] \hat{\mu}_2(r, \mu) dr < .9K$ .

The maximum value  $Q$  can take is the value of exerting minimum effort (0) and holding office forever:  $\frac{u(0)}{1-\delta}$ . Thus, because

$\left| \int_{r(\mu)}^{\infty} f_{aa}(r|a(\mu)) dr \right| < B$  for some  $B > 0$  I can conclude that  $\left| \int_{r(\mu)}^{\infty} a'(\mu) Q(\hat{\mu}(r, \mu)) f_{aa}(r|a(\mu)) dr \right| < B \frac{v}{\mu_0^2} \frac{u(0)}{1-\delta}$  where  $v$  is determined by the choice of  $r(\mu)$  made above.

Similarly, by assumption  $|u''(a(\mu))| < \infty$ . I may focus on a closed interval  $a \in [0, \frac{v}{\mu_0}]$  so that the second derivative is uniformly bounded above:

$$|u''(a(\mu))| < U \text{ for some } U > 0.$$

Using these bounds, I have that the absolute value of the derivative above is bounded by:

$$0.9K + \frac{f(r(\mu)|a(\mu))}{f_a(r(\mu)|a(\mu))} \frac{v}{\mu_0^2} \left( U + B \frac{u(0)}{1-\delta} \right) < K$$

The first term is strictly less than  $K$ . The second term does not depend on  $K$ , so that choosing  $K$  high enough makes it strictly less than  $0.1K$ . ■

Having a bounded derivative also ensures that the class of functions  $T(\hat{C})$  is

equicontinuous. A class of functions is equicontinuous if, given  $\varepsilon > 0$ , there is a  $\delta > 0$  such that  $|f(x) - f(y)| < \varepsilon$  whenever  $|x - y| < \delta$  for any  $x$  in the domain of  $f$  and any  $f \in T(\hat{C})$ .

**Lemma 3.** *Let  $T(\hat{C})$  be a class of bounded, continuous and differentiable functions with a uniformly bounded derivative. Then  $T(\hat{C})$  is equicontinuous.*

**Proof.** For any  $f \in T(\hat{C})$ ,  $\frac{|f(x) - f(y)|}{|x - y|} \approx f'(x)$ . Because  $|f'(x)| < B$ ,  $|f(x) - f(y)| < B|x - y|$ .

Because the bound  $B$  on the derivative is the same for all  $f \in T(\hat{C})$ , if we choose  $x$  and  $y$  such that  $|x - y| < \frac{\varepsilon}{B}$ ,  $|f(x) - f(y)| < \varepsilon$  for any  $f \in T(\hat{C})$ . Therefore  $T(\hat{C})$  is equicontinuous. ■

Next I verify that the operator  $T$  is continuous.

**Lemma 4.** *The operator  $T$  is continuous.*

**Proof.** Let  $\{Q_i\}_{i \in \mathbb{N}} \subset \hat{C}$  be a sequence of functions converging to  $Q$  in the sup norm.

Then, for any  $\beta > 0 \exists j \in \mathbb{N}$  such that  $\forall i > j, \|Q_i - Q\| < \beta$ .

$$(T(Q_i) - T(Q))(\mu) = \delta \int_{r_Q(\mu)}^{\infty} [Q_i(\hat{\mu}(r, \mu)) - Q(\hat{\mu}(r, \mu))] f(r|a(\mu)) dr \\ + \delta \int_{r_{Q_i}(\mu)}^{r_Q(\mu)} Q_i(\hat{\mu}(r, \mu)) f(r|a(\mu)) dr$$

if  $r_{Q_i}(\mu) > r_Q(\mu)$ . For the reverse case, an identical argument may be used.

The first term converges to zero by definition of  $Q_i$ .

The second term converges to zero because  $r_{Q_i}(\mu) \rightarrow r_Q(\mu)$ . To see this, consider the following equality derived from the politician's F.O.C.:

$$\int_{r_Q(\mu)}^{\infty} [Q_i(\hat{\mu}(r, \mu)) - Q(\hat{\mu}(r, \mu))] f_a(r|a(\mu)) dr = \int_{r_{Q_i}(\mu)}^{r_Q(\mu)} Q_i(\hat{\mu}(r, \mu)) f_a(r|a(\mu)) dr$$

Again, the term on the LHS converges to zero by convergence of  $Q_i$ . Hence the RHS must also converge to zero. However, because  $r_{Q_i}(\mu) > a(\mu)$  and  $Q_i(\hat{\mu}(r, \mu)) \geq u(0)$ , the terms inside the integral are bounded away from zero. Therefore, it must be that  $r_{Q_i}(\mu) \rightarrow r_Q(\mu)$ .

I have now established that  $\|T(Q_i) - T(Q)\| \rightarrow 0$  so that  $T$  is a continuous operator. ■

I may now apply Schauder's FPT to find a value function and a reputation-dependent cutoff function  $r(\mu)$  implementing effort strategy  $a(\mu, v)$ .

This completes the proof of existence.

## 7.2. Impossibility of Markov perfect equilibria with positive effort - proof of Proposition 2

In this section I present a proof of a slightly more general version of Proposition 2. Specifically, I generalize the statement to include strategies which are Markovian with probability 1.

**Definition 8.** *An equilibrium is essentially Markov perfect if strategies  $(\sigma, a)$  are measurable with respect to the Markovian partition for a set of reputations  $M \subset [0, 1]$  of Lebesgue measure 1.*

Note that any Markovian strategy is also essentially Markovian. Although the distinction is not of interest in and of itself, I make it here as it is useful in establishing Proposition 3 in Section 4. The extension does not significantly complicate the proof since it requires only that we note that non-Markovian strategies which are played with probability 0 do not affect the strategic calculus of players involved.

**Proposition 4.** *There is no essentially Markov perfect equilibrium with positive value for the voter.*

In what follows, for ease of exposition I write  $\hat{Q}(\hat{\mu}(r, \mu))$  for  $\sigma(\hat{\mu}(r, \mu))Q(\hat{\mu}(r, \mu))$ .

The proof proceeds as follows. First, I consider the case in which effort is bounded below for some interval  $[m, 1]$  of reputations and  $\hat{Q}$  is weakly monotonic. This leads me to conclude that  $\hat{Q}$  is unbounded, a contradiction.

Then, I generalize the result in several ways. First, if  $\hat{Q}$  is not weakly monotonic, I show that one may look at a moving average of  $\hat{Q}$  and that repeated application of the moving average operator leads to a function which is monotonic or

approximately constant over an interval  $[z, 1)$ , and thus to the same contradiction as above.

Once this is done, I am left with the possibility that effort is not bounded below. However, I show that, if positive effort is ever incentivized, politicians with high reputation must be reelected with positive probability and, if that is the case, there must be politicians of arbitrarily high reputation who exert effort above some fixed lower bound. Therefore, I am able to complete the argument by showing that these minimum conditions are enough to lead to the conclusion that  $\hat{Q}$  is unbounded. Thus, there can be no Markov perfect equilibrium supporting positive effort if the politician's payoffs are bounded.

Consider first the case in which there is a lower bound  $b > 0$  on the effort exerted by politicians with reputation in  $[x, 1)$ . Using the politician's FOC, I know that his value function must satisfy

$$\delta \int_{-\infty}^{\infty} \hat{Q}(\hat{\mu}(r, \mu)) f_a(r|a(\mu)) dr \geq -u'(b) = B > 0$$

By A4. I can rewrite  $\hat{Q}(\hat{\mu}(r, \mu)) f_a(r|a(\mu))$  as  $\hat{Q}(\hat{\mu}(r + a(\mu), \mu)) f_a(r|0)$ .

Because  $\int_0^{\infty} f_a(r|0) dr < \infty$ , I can find a value  $r^* \in \mathbb{R}_+$  such that

$$\delta \int_{-r^*}^{r^*} \hat{Q}(\hat{\mu}(r + a(\mu), \mu)) f_a(r|0) dr \geq \delta \int_{-\infty}^{\infty} \hat{Q}(\hat{\mu}(r + a(\mu), \mu)) f_a(r|0) dr - \varepsilon$$

for some fixed  $\varepsilon \in (0, \frac{B}{2})$ .

Suppose  $\hat{Q}$  is weakly monotonic. If  $\hat{Q}$  is weakly decreasing, the integrals above will be weakly negative since, by A5.,  $\delta \int_{-\infty}^{\infty} \hat{Q}(\hat{\mu}(r + a(\mu), \mu)) f_a(r|0) dr = \delta \int_0^{\infty} [\hat{Q}(\hat{\mu}(r + a(\mu), \mu)) - \hat{Q}(\hat{\mu}(-r + a(\mu), \mu))] f_a(r|0) dr < 0$ . Thus, the F.O.C. will not be satisfied. Suppose  $\hat{Q}$  is weakly increasing. By the monotone likelihood ratio property (A3.), I know that there is a unique point at which  $f_a(r|0) = 0$  with the derivative being negative to the left and positive to the right of that point. Because  $f(r|0)$  is symmetric (A5.), this point is 0. Then,

$$\begin{aligned} & \delta \int_{-r^*}^{r^*} \hat{Q}(\hat{\mu}(r + a(\mu), \mu)) f_a(r|0) dr \\ & \leq \delta \int_0^{r^*} \hat{Q}(\hat{\mu}(r^* + a(\mu), \mu)) f_a(r|0) dr + \delta \int_{-r^*}^0 \hat{Q}(\hat{\mu}(-r^* + a(\mu), \mu)) f_a(r|0) dr \\ & \leq \delta [\hat{Q}(\hat{\mu}(r^* + a(\mu), \mu)) - \hat{Q}(\hat{\mu}(-r^* + a(\mu), \mu))] k \end{aligned}$$

where  $k = \int_0^{r^*} f_a(r|0)dr$ .

Therefore,  $\hat{Q}(\hat{\mu}(r^* + a(\mu), \mu)) - \hat{Q}(\hat{\mu}(-r^* + a(\mu), \mu)) \geq \frac{B}{2\delta k} > 0$  for all  $\mu$ .

Given  $\mu$  and  $r^*$ , there is a  $\mu'$  such that  $\mu = \hat{\mu}(-r^* + a(\mu'), \mu')$ . Therefore,  $\hat{Q}$  must increase by at least  $\frac{B}{2\delta k}$  over  $[\hat{\mu}(-r^* + a(\mu'), \mu'), \hat{\mu}(r^* + a(\mu'), \mu')]$ . Because this process can be repeated indefinitely, this implies that  $\hat{Q}$  grows without bound, which is a contradiction. Therefore, there can be no Markov reelection strategy leading to a weakly monotonic  $\hat{Q}$  over any interval  $[x, 1]$  while effort is bounded below by  $b > 0$ .

I am left with the possibility of a  $\hat{Q}$  which is non-monotonic over every interval of the form  $[x, 1]$ . Suppose I have found such a  $\hat{Q}$ . Then,

$$\delta \int_{-r^*}^{r^*} \hat{Q}(\hat{\mu}(r + a(\mu), \mu)) f_a(r|0) dr \geq \frac{B}{2} \text{ for all } \mu.$$

Define  $\hat{Q}(x) = \hat{Q}(\hat{\mu}(r + a(x), x))$  for  $x \in [m, 1]$ . Then,

$$\delta \int_{-r^*}^{r^*} \hat{Q}(x) f_a(r|0) dr \geq \frac{B}{2}$$

Therefore,  $\delta \int_{-r^*}^{r^*} \frac{1}{\hat{\mu}(r^*, \mu) - \mu} \int_{\mu}^{\hat{\mu}(r^*, \mu)} \hat{Q}(x) dx f_a(r|0) dr \geq \frac{B}{2}$

$\frac{1}{\hat{\mu}(r^*, \mu) - \mu} \int_{\mu}^{\hat{\mu}(r^*, \mu)} \hat{Q}(x) dx$  is a moving average of  $\hat{Q}$ . We may apply this operator repeatedly defining  $\hat{Q}_0 = \hat{Q}$  and  $\hat{Q}_i(\mu) = \frac{1}{\hat{\mu}(r^*, \mu) - \mu} \int_{\mu}^{\hat{\mu}(r^*, \mu)} \hat{Q}_{i-1}(x) dx$ . The following Lemma establishes a basic but useful fact about the moving average operator.

**Lemma 5.** *Given a function  $\hat{Q}$ , there exists an interval of positive length  $[z, 1)$  such that  $\hat{Q}_2$  is either weakly monotonic or approximately constant on  $[z, 1)$ .*

**Proof.** After the moving average operator has been applied once,  $\hat{Q}_1$  is continuous and differentiable with derivative

$$\hat{Q}'_1(\mu) = \frac{\partial(\frac{1}{\hat{\mu}(r^*, \mu) - \mu})}{\partial \mu} \int_{\mu}^{\hat{\mu}(r^*, \mu)} \hat{Q}_0(x) dx + \frac{1}{\hat{\mu}(r^*, \mu) - \mu} \left( \hat{\mu}_2(r^*, \mu) \hat{Q}_0(\hat{\mu}(r^*, \mu)) - \hat{Q}_0(\mu) \right).$$

Therefore  $\hat{Q}_2$  is continuously differentiable and

$$\hat{Q}'_2 = \frac{\partial(\frac{1}{\hat{\mu}(r^*, \mu) - \mu})}{\partial \mu} \int_{\mu}^{\hat{\mu}(r^*, \mu)} \hat{Q}_1(x) dx + \frac{1}{\hat{\mu}(r^*, \mu) - \mu} \left( \hat{\mu}_2(r^*, \mu) \hat{Q}_1(\hat{\mu}(r^*, \mu)) - \hat{Q}_1(\mu) \right).$$

$$\begin{aligned} \hat{Q}''_2 &= \frac{\partial^2(\frac{1}{\hat{\mu}(r^*, \mu) - \mu})}{\partial \mu^2} \int_{\mu}^{\hat{\mu}(r^*, \mu)} \hat{Q}_1(x) dx + 2 \frac{\partial(\frac{1}{\hat{\mu}(r^*, \mu) - \mu})}{\partial \mu} \left( \hat{\mu}_2(r^*, \mu) \hat{Q}_1(\hat{\mu}(r^*, \mu)) - \hat{Q}_1(\mu) \right) \\ &+ \frac{1}{\hat{\mu}(r^*, \mu) - \mu} \left( \hat{\mu}_{22}(r^*, \mu) \hat{Q}_1(\hat{\mu}(r^*, \mu)) + \hat{\mu}_2(r^*, \mu) \hat{Q}'_1(\hat{\mu}(r^*, \mu)) - \hat{Q}'_1(\mu) \right). \end{aligned}$$

Because  $\hat{Q}$  is bounded,  $\hat{Q}'_2$  and  $\hat{Q}''_2$  are bounded. Let  $B > 0$  denote the bound on  $\hat{Q}''_2$ .

Given an  $\varepsilon > 0$ , there is a  $z$  such that if  $|\hat{Q}'_2(\mu)| > \varepsilon$  for some  $\mu \in [z, 1)$  then  $\hat{Q}_2$  is strictly monotonic over  $[z, 1)$ . This is because the most  $\hat{Q}'_2$  can change in a distance less than  $1 - z$  is  $B(1 - z) < \varepsilon$  for  $z$  close enough to 1. If there is no  $\mu \in [z, 1)$  such that  $|\hat{Q}'_2(\mu)| > \varepsilon$ , then  $\|\hat{Q}_2 - C\|_\infty < \eta$  for some constant function  $C$  and a  $\eta$  which becomes arbitrarily small as  $\varepsilon \rightarrow 0$ . Thus,  $\hat{Q}_2$  is approximately constant. ■

If  $\hat{Q}_2$  is weakly monotonic over  $[z, 1)$ , I may now repeat the arguments for weakly monotonic functions on  $\hat{Q}_2$  starting at the point  $z$ . Since a bounded  $\hat{Q}$  should imply a bounded  $\hat{Q}_2$ , I am once again left with a contradiction. If  $\hat{Q}_2$  is merely approximately constant, I note that  $\delta \int_{-r^*}^{r^*} C f_a(r|0) dr = 0$  by symmetry of  $f(r|0)$  (A5.) and, for  $\mu$  such that  $\hat{\mu}(-r^* + a(\mu), \mu) > z$ ,

$$\begin{aligned} & \left| \int_{-r^*}^{r^*} \hat{Q}_2(\hat{\mu}(r + a(\mu), \mu)) f_a(r|0) dr - \int_{-r^*}^{r^*} C f_a(r|0) dr \right| \\ &= \left| \int_{-r^*}^{r^*} \hat{Q}_2(\hat{\mu}(r + a(\mu), \mu)) f_a(r|0) dr \right| < \frac{B}{2} \text{ (if } \eta \text{ is chosen small enough) which} \\ & \text{contradicts the derived properties of } \hat{Q}. \end{aligned}$$

Now, I consider the case where there is no lower bound on effort exerted. The following Lemmas provide constraints on what can happen in such a hypothetical equilibrium.

**Lemma 6.** *In any Markov equilibrium with positive value  $V(\mu_0) > 0$ , every interval of the form  $[\mu, 1]$  must contain reputation points at which politicians are reelected with strictly positive probability.*

**Proof.** Suppose not. Let  $\hat{r}(a)$  denote the outcome which would keep the politician's reputation constant:

$$\hat{r}(a) = \{r | \hat{\mu}(r, \mu) = \mu\}$$

Note that, using assumption A3.,  $\hat{r}(a) < a$  (if  $r$  is normally distributed  $\hat{r}(a) = \frac{a}{2}$ ).

Then consider the first order condition of a politician with the *highest reputation which is reelected with positive probability*  $\mu$ :

$$u'(a(\mu)) + \delta \int_{-\infty}^{\hat{r}(a)} Q(\hat{\mu}(r, \mu)) f_a(r|a(\mu)) dr < 0 \text{ for any } a(\mu).$$

Because  $f_a(r|a(\mu))$  is negative for all values below  $a(\mu)$ . Therefore,  $a(\mu) = 0$  and  $\mu$  is an absorbing state. Since I assumed  $V(\mu_0) > 0$ , it is not a best response for the voter to reelect a politician with reputation  $\mu$ , contradicting the definition of  $\mu$ . ■

**Lemma 7.** *Consider a Markov perfect equilibrium with positive value for the voter  $V(\mu_0) > 0$ . In every reputation interval of the form  $[\mu, 1]$  there must be a subset of positive measure in which politicians exert effort above some fixed lower bound  $b > 0$ .*

**Proof.** Suppose not. Then, choose a lower bound  $b < \frac{1}{2}V(\mu_0)$  and let  $[\mu, 1]$  be an interval over which effort is bounded above by  $b$  almost everywhere.  $V$  is bounded above by the constant function  $\bar{V} = \frac{\bar{a}}{1-\delta}$  where  $u(\bar{a}) = 0$ . Let  $k$  satisfy  $\sum_{i=0}^k \delta^i b + \sum_{i=k+1}^{\infty} \delta^i \bar{V} < V(\mu_0)$ . By Lemma 6, there must be reputations arbitrarily close to 1 which are reelected with positive probability. Because effort is bounded, I may choose a reputation (call it  $\hat{\mu}$ ) which is reelected with positive probability and from which the probability of transitioning out of  $[\mu, 1]$  in  $k$  periods or fewer (call it  $p$ ) is arbitrarily small. In particular, if I choose  $p < \frac{V(\mu_0)}{\delta \bar{V}}$ , an upper bound on the value to the voter of having a politician with reputation  $\hat{\mu}$  in office ( $V(\hat{\mu})$ ) is:

$$V(\hat{\mu}) < (1 - p) \left( \sum_{i=0}^k \delta^i b + \sum_{i=k+1}^{\infty} \delta^i \bar{V} \right) + p \delta \bar{V} < V(\mu_0)$$

If the politician is reelected in each of his first  $k$  terms. Note that the probability of transitioning to a point in  $[\mu, 1]$  at which effort higher than  $b$  is exerted is

zero because this may happen only on a subset of measure 0, and therefore this possibility does not affect the calculation of expected rewards.

If he does not survive  $k$  terms, then  $V(\hat{\mu})$  is less than:

$$V(\hat{\mu}) < b + \delta V(\mu_0) < V(\mu_0)$$

Therefore, it is not a best response to reelect a politician when his reputation is  $\hat{\mu}$ , which contradicts the definition of  $\hat{\mu}$ . ■

Given Lemma 7, if I have a weakly monotonic value function I need only to modify the arguments above as follows. Instead of moving to a reputation satisfying  $\mu = \hat{\mu}(-r^* + a(\mu'), \mu')$  I move to one satisfying  $a(\mu') > b$  and  $\mu < \hat{\mu}(-r^* + a(\mu'), \mu')$ . Once again, I conclude that  $\hat{Q}$  must increase by at least a fixed amount  $\frac{B}{2\delta k}$  infinitely many times, contradicting its boundedness.

To deal with non-monotonic candidate value functions  $\hat{Q}$  I note that, given Lemma 7, repeated application of the moving average operation ensures that the value of all integrals  $\delta \int_{-r^*}^{r^*} \hat{Q}_i(\hat{\mu}(r, \mu)) f_a(r|a(\mu)) dr$  will be positive. Because these are defined on a closed set  $[\mu', 1]$ , there exists a minimum value of these integrals. Now, I may apply the same arguments as above:  $\hat{Q}_2$  includes a weakly monotonic segment  $[z, 1)$ , and this contradicts the boundedness of  $\hat{Q}$ .

Finally, note that in all the arguments above, having a function  $Z$  which differs from  $\hat{Q}$  only on a set of Lebesgue measure 0 will not change any of the results, because the integrals will yield the same values under both functions. Therefore, it is immediate that the result extends to rule out essentially Markov perfect equilibria with positive value for the voter.

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