

Reputation and Accountability in Repeated Elections*

Rainer Schwabe[†]

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Abstract

This paper studies a model of infinitely repeated elections in which voters attempt simultaneously to select competent politicians and to provide them with incentives to exert costly effort. When voters base their reelection decisions only on incumbent reputation, incentives for effort are ineffective. However, equilibria in which politicians of all reputations work and voters use reputation-dependent performance cutoffs (RDC) to make reelection decisions exist. In these equilibria, politicians' effort is decreasing in reputation, and expected performance is decreasing in tenure. Like the equilibria in Ferejohn (1986), RDC equilibria rely on voters being indifferent between reelecting incumbents and electing challengers. I show that this voter-indifference condition is closely related to weak renegotiation-proofness (Farrell and Maskin (1989)).

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[†]Banco de México. Email: rschwabe@banxico.org.mx

1. Introduction

In a representative democracy, voters have the power to choose which citizens will occupy government posts. Even if they cannot directly observe politicians' actions, voters may harness this power to induce incumbents to work in their interest by conditioning reelection on performance. This understanding of the relationship between voter and politician (Key (1966), Barro (1973), Ferejohn (1986)) is the driving force behind all models of political agency.

If, as seems certain, politicians differ in their ability or preferences, an additional consideration must be taken into account by voters. There is a trade-off between having a reelection rule which effectively aligns the interests of the incumbent with the voters', and one which focuses on reelecting the type of politicians who are most able or willing to work in the voters' interest. In order to provide incentives, the voter must sometimes throw capable politicians out of office. These dual roles of elections are commonly referred to as sanctioning and selection.¹

The relative importance of these roles is a theoretical question which will have different answers depending on the institutional environment (e.g. are there term limits?) and the type of heterogeneity among politicians which is considered (e.g. do they differ in honesty or competence?). The main goal of this paper is to contribute to the understanding of the twin roles of elections, how voters may optimally and credibly balance them, and the implications of this balance for voter behavior and political careers through time.

Most work in this area has concluded that voters will focus exclusively on selection. In an influential article, Fearon (1999, p. 77) states that "when it comes time to vote it makes sense for the electorate to focus completely on the question of type: which candidate is more likely to be principled and share the public's preferences?" Similar claims are made by Besley (2006) and others (see the related literature section below) whose models predict that, when making reelection deci-

¹See Section 4 for a discussion of the relation between sanctioning and selection, and retrospective and prospective voting.

sions, voters will consider only the incumbent’s probability of being a ‘good’ type. While this conclusion has some force, it is a consequence of modelling choices such as the use of two-period models and the treatment of ability as a substitute rather than a complement to effort.²

The popularity of these modelling choices is, at least partly, a consequence of the difficulties involved in analyzing models with no term limits and heterogeneity in politicians’ cost of effort. One of the earliest and most influential articles to study political agency with moral hazard and adverse selection (Banks and Sundaram (1993)) chose this framework. However, the difficulty of the analysis and the opportunities presented by other modeling alternatives discouraged follow-up work.³ In this paper, I retake this road and propose a class of equilibria which are analytically simple yet offer rich predictions for the evolution of voter behavior and political careers. In contrast to previous work, I find that sanctioning must be a central component of voter behavior in any equilibrium in which rent-extraction is limited.

I study an infinite-horizon model of repeated elections with no term limits in which politicians differ in their competence. Competent politicians can improve the expected utility of voters by exerting costly effort (a). They may also provide utility $\lambda \geq 0$ to voters independent of their effort choice, making voter utility $\lambda + a + \varepsilon$ where ε is a noise term. Incompetent politicians, on the other hand, do not have the ability to improve outcomes, or it is prohibitively costly for them to do so, so that voter utility is equal to ε . My focus is on small values of λ , so that the main difference between competent and incompetent politicians is the ability to improve voter utility through effort. The voters’ beliefs about the likelihood of an incumbent being competent evolve along with his observed performance. These beliefs are the politician’s reputation.

²The most common specification of voter utility in the literature is $\lambda + a + \varepsilon$ where λ is incumbent type, a is effort, and ε is a noise term. In this paper, I use the same specification but focus on the case in which λ is small.

³In Banks and Sundaram (1998), term limits and the direct effect of type on voter utility are introduced, leading to a richer set of predictions.

Having set up the model, I present formal definitions of sanctioning and selection based on the information used in voters' reelection strategies. The selection-only criterion coincides with Markov perfection. In the first of this paper's main results (Theorem 1), I show that in any Markov perfect equilibrium the voters' payoffs are bounded above by λ . Sanctioning-only equilibria, in which the voters reelect incumbents based only on their recent performance, involve high levels of incumbent effort. However, the voters face a commitment problem which undermines the credibility of their reelection strategy, that is, these equilibria are not weakly renegotiation-proof (WRP, Farrell and Maskin (1989)).

I go on to define and prove existence of equilibria which are WRP and give the voter expected utility higher than λ (Theorem 2). In reputation-dependent performance threshold (RDC) equilibria, incumbents are reelected only if their observed performance exceeds a cutoff which varies with their reputation. These performance cutoffs vary in such a way as to make it incentive compatible for politicians to exert just enough effort to leave voters indifferent between reelecting the incumbent and electing a challenger, thus making the voters' value function constant across reputations. This voter indifference is what makes the voters' reelection strategy credible. Indeed, voter indifference and WRP are qualifiedly equivalent (Theorem 3). This result not only strengthens the argument for RDC equilibrium, it also attests to the robustness of the equilibria in Ferejohn (1986), the most widely used political agency model, to the introduction of heterogeneity among politicians.

In RDC equilibria, politicians with high reputation exert low effort. Also, in expectation, reputation is positively related to tenure so that, for a given politician, tenure is negatively related to performance (see Proposition 3). This provides a novel explanation for the negative correlation between tenure and constituency service in the U.S. House of Representatives (Cain, Ferejohn, and Fiorina (1990), Ashworth (2005)) that is not based on last-period effects.

The paper proceeds as follows. In the following subsection, I discuss related work and its this paper's place in the literature. In Section 2, I describe the model

and its assumptions. Section 3 addresses the problem of multiplicity of equilibria and introduces equilibrium selection criteria. In Section 4, I define sanctioning and selection and look at equilibria involving only one of the roles of elections. In Section 5, I define RDC equilibrium and prove its existence. Section 5.1 asks what RDC equilibria can tell us about the dynamics of political careers. Section 6 examines the relation between voter indifference and WRP. Section 7 concludes.

1.1. Related Literature

There is a growing number of papers which study the sanctioning and selection roles of elections in a unified framework. Much of this work builds on work by Holmström (1999) on career concerns, with the relationship most directly apparent in Persson and Tabellini (2000, ch. 4.5). Notable contributions include Reed (1994), Banks and Sundaram (1998), Fearon (1999), Berganza (2000), Ashworth (2005), and Besley (2006, ch. 3.3). Each of these works studies a model in which voters consider both the selection and incentivizing roles of elections and politicians face term limits. Additionally, several papers have applied similar models to the study of subjects such as transfers to special interest groups (Coate and Morris (1995) and Lohmann (1998)), the incumbency advantage (Ashworth and Bueno de Mesquita (2006)), constituency service (Ashworth and Bueno de Mesquita (2008)), and CEO activism (Dominguez-Martinez, Swank, and Visser (2008)) to name a few.

There are two important differences between the models cited in the previous paragraph and this paper. First, imposing term limits means that last period behavior is easily solved for, and reelection rules are derived by backward induction. In this paper there are no term limits, so voters and politicians face a dynamic problem at every stage. The second is the type of politician heterogeneity studied. In the papers above, voters are assumed to benefit from having a high type in office even if the he will exert no effort. While I allow for this type of heterogeneity, high types differ from low types primarily in their *ability* to induce outcomes preferred by the voters through costly effort. I feel that this is a more natural way

of modeling differences in ability or competence, while the alternative approach is best suited to modeling differences in honesty or alignment of preferences.

Bobonis, Cámara Fuertes, and Schwabe (2011) study the effect of publicly released audits of municipal governments in Puerto Rico on political corruption and voting behavior. As expected, audits keep incumbents honest in the short-term. While audits do not predict future corruption, they do have a positive effect on future reelection rates. An adaptation of the model in this paper explains these regularities. Furthermore, the authors argue that the combination of positive selection effects but nil dynamic performance effects observed in the data is particular to RDC equilibria of agency models in which politicians differ in their competence but not their honesty.

In a closely related paper, Banks and Sundaram (1993) study the selection and sanctioning roles of elections in a fully dynamic framework with no term limits. This paper differs from their's mainly in terms of equilibrium selection, although this leads to important differences in predictions made about political careers and in insights gained into the interplay of the two roles of elections. I discuss the differences in greater detail in Section 4.2.

Duggan (2000) and Banks and Duggan (2008) study a model of repeated elections in which politicians differ in their spatial policy preferences. Voters use the incentive of reelection to induce politicians to temper their policy choices while in office. However, because there is no uncertainty in the execution of policy and strategies are stationary, there is no evolution of beliefs about the incumbent's preferences beyond their first period in office.

Meirowitz (2007) proposes a model of repeated elections in which two long-lived parties, differing in their policy preferences and valence, compete in elections each period. Voters are uncertain about the set of feasible policies rather than about the parties' characteristics or the policy choices made. He shows that, while electoral control is impossible if voters are constrained to using pure strategies, perfect control is possible in mixed strategies. If mixed strategies are to be used, each party must provide the same expected utility to the voter when in office.

This leads to a voter indifference condition analogous to the one emphasized in this paper.

Smart and Sturm (2006) present a model of repeated elections in which incumbents' actions are publicly observable, but the underlying state of the world which determines which action is good for the voters is observed only by the incumbent. In this context, they prove that the best Markov perfect equilibrium in the absence of term limits involves all politicians taking the same action regardless of the state of the world. They go on to argue that imposing term limits may help voters by decreasing the incentives for politicians to conform. Their result on the limits of Markov perfect equilibria are in the spirit of the first main result of this paper. In this paper, voters are better able to control their representatives because they condition reelection on their own utility, which contains information about incumbent's actions.

This paper is related to the literature on dynamic principal-agent interactions outside of the political sphere. The approach taken here differs from that taken in much that literature in two main dimensions. First, this paper focuses on the use of a retention rule rather than a compensation contract as an incentivizing mechanism. Second, in most of the literature on principal-agent relationships the principal is assumed to be a Stackelberg first-mover, leaving the agent only his reservation utility. In this paper, I look at Nash equilibria which admit the possibility that the gains from interaction may be shared. Indeed, in the RDC equilibria which I focus on, the agent reaps all of the benefits from increases in his reputation and enjoys utility strictly greater than his reservation value.

Mailath and Samuelson (2001) and Hörner (2002) study related models of reputation formation in which firms attempt to convince consumers that they are competent. Consumers are willing to pay more for a competent firm's products only if they expect the firm to exert high effort. In a result reminiscent my Theorem 1, Mailath and Samuelson show that, with persistent types, Markov perfect equilibria cannot support high effort. They share my skepticism of trigger strategy equilibria but, rather than using a renegotiation-proofness argument to

make this point, they argue instead for a restriction to Markovian strategies. They show that effort can be incentivized if a firm’s type changes with some positive probability every period so that reputation cannot become ‘too good’. Hörner studies a similar model in which many firms compete with each other for marketshare while developing a reputation for competence. He shows that, even with persistent types, effort can be incentivized if firms believe that customers will leave them for a competitor after a bad outcome. Intriguingly, his equilibria involve customers being left indifferent among firms of varying reputations as high reputation firms charge higher prices. However, this is assumed as an equilibrium condition and supported by appropriate beliefs off the equilibrium path. While the equilibria in Hörner’s model are Markov perfect, this relies on having only two possible outcomes so that reputations correspond to particular histories of play. Having continuous outcomes would make this partition impossible and make clear that his equilibria have a similar strategic complexity to the RDC equilibria studied in this paper.

2. The Model

I study a discrete-time, infinite horizon model of a democratic society. In order to focus on the problems of selecting competent politicians and providing them with incentives to perform well, I abstract from ideological differences in the electorate. Instead, I model citizens as a single, infinitely-lived representative voter.

2.1. Preferences, Timing, and Information in the Stage Game

Each period, indexed by $t \in \{1, 2, \dots\}$, the voter must select a politician to carry out a task. There is an infinite set P of potential politicians from which challengers are randomly drawn. Each politician is infinitely-lived and may serve for as many periods, or terms, in office as the voter asks him to. Once replaced by a challenger,

however, a politician cannot return to office.⁴

Politicians are one of two types: competent (CP) or incompetent (IP). Competent politicians exert effort $a \in \mathbb{R}_+ = [0, \infty)$. This effort impacts, but does not perfectly determine, the voter's stage-game utility $r \in \mathbb{R}$. Specifically,

$$r = \lambda + a + \varepsilon \tag{2.1}$$

where $\lambda \geq 0$ is a fixed benefit of having a competent incumbent and ε is a zero-mean noise term. Effort is related to voter utility via an effort-contingent distribution function $F(r|a)$ with density $f(r|a)$:

$$E(r|a, CP) = \int_{-\infty}^{\infty} r f(r|a) dr = \lambda + a + \int_{-\infty}^{\infty} \varepsilon f(\varepsilon | -\lambda) d\varepsilon = \lambda + a \tag{2.2}$$

CPs receive per-period utility $u(a)$ when in office, and 0 otherwise. The utility function $u(a)$ is twice continuously differentiable and strictly concave. Effort is costly so that u is weakly decreasing in a with $u'(0) = 0$ and $u'(a) < 0 \forall a > 0$. I also assume that $u(a) > 0$ for all $a \in [0, \bar{a})$ and some $\bar{a} > 0$. In order to ensure that the incumbent's utility maximization problem has a unique solution characterized by a first order condition, I assume that the function u is sufficiently concave (u'' is sufficiently negative).⁵

Incompetent politicians are unable⁶ to affect the distribution of r so that it is always

$$F_{IP}(r) = F(r | -\lambda) \tag{2.3}$$

when an incompetent politician is in office. They receive a payoff $u_{IP} > 0$ when in office and 0 otherwise, so that they are always willing to serve if elected. Because

⁴Allowing for politicians to return to office can be useful in applications. See Svobik (2010).

⁵I leave the specifics of this assumption to the Appendix (Assumption A5.).

⁶Alternatively, effort is too costly for L-types for it to be worthwhile exerting. $-u'_L(0) > \frac{\delta}{1-\delta} u_L(0) f(0 | -\lambda)$ is sufficient for this if I make the same assumptions on u_L as on u .

IPs are always willing to hold office but cannot make choices which influence payoffs, the analysis will center on competent politicians' choices.

As is standard in games with imperfect monitoring (Abreu, Pearce, and Stacchetti (1990)), I assume that the distribution of results has full support: $f(r|a) > 0$ for all r and a . This guarantees that effort level and type can never be perfectly inferred by observing results. I also make the following assumptions for analytical convenience. First, that $f(r|a)$ is twice continuously differentiable in both arguments. Second, $f(r|a)$ is symmetric around its mean.

Because the evolution of beliefs about incumbent types is central in this paper, it is useful to make assumptions ensuring that good results are more likely when effort is high. Thus, I assume that $f(r|a)$ satisfies the monotone likelihood ratio property (Milgrom (1981)): $\frac{f(x|a)}{f(x|a')} > \frac{f(y|a)}{f(y|a')}$ whenever $x > y$ and $a > a'$.

A politician's type is the private information of the politician. The voter believes the incumbent to be competent with some probability μ ; the incumbent's *reputation*. Note that the expected stage-game payoff to the voter when a competent incumbent exerts effort a is $\mu(\lambda + a)$, so that reputation is payoff relevant. The proportion of competent types among the set of potential politicians P is μ_0 . Because new politicians are selected randomly from this set, μ_0 will also be the reputation of any politician at the beginning of his first term.

2.2. Histories, Strategies, and the Repeated Game

At time t , the voter and all politicians have information about who has been in office and what rewards the voter has received in all previous periods $(1, 2, \dots, t-1)$. A realization of these variables is a public t-history, labeled h_t . H denotes the set of all possible public t-histories.

A *reelection strategy* is a function $\sigma : H \rightarrow [0, 1]$ denoting the probability with which the voter will reelect the incumbent, conditional on all currently available information.

Similarly, an *effort strategy* for competent politician i is a function $\alpha_i : H \rightarrow \mathbb{R}_+$ denoting the effort which a given politician will exert conditional on being in

function. Letting $h_{t+1}(r)$ denote the $t+1$ -history reached from h_t ⁷ after a result r is observed, it may be defined recursively:

$$\begin{aligned} V(\sigma, \alpha, \hat{\mu}; h_t) &= \hat{\mu}(h_t) (\lambda + \alpha(h_t)) + \delta \hat{\mu}(h_t) \int_{-\infty}^{\infty} V(\sigma, \alpha, \hat{\mu}; h_{t+1}(r)) f(r|\alpha(h_t)) dr \\ &\quad + \delta (1 - \hat{\mu}(h_t)) \int_{-\infty}^{\infty} V(\sigma, \alpha, \hat{\mu}; h_{t+1}(r)) f_{IP}(r) dr \end{aligned} \quad (2.5)$$

where $\delta \in (0, 1)$ is a discount factor common to the voter and all politicians.

Similarly, I denote the value function of a competent incumbent $Q(\sigma, \alpha, \hat{\mu}; h_t)$. It may be defined recursively as:

$$Q(\sigma, \alpha, \hat{\mu}; h_t) = u(\alpha(h_t)) + \delta \int_{-\infty}^{\infty} Q(\sigma, \alpha, \hat{\mu}; h_{t+1}(r)) f(r|\alpha(h_t)) dr \quad (2.6)$$

Note that I do not explicitly write the reelection probability $\sigma(h_{t+1}(r))$ in the value functions above. Instead, $h_{t+1}(r)$ captures whether the incumbent is reelected, in which case $Q(\sigma, \alpha, \hat{\mu}; h_{t+1}(r)) > 0$, or a challenger with reputation μ_0 is elected and $Q(\sigma, \alpha, \hat{\mu}; h_{t+1}(r)) = 0$.

Definition 1. *A perfect public equilibrium is a strategy profile (σ^*, α^*) and a belief function $\hat{\mu}$ such that:*

1. $V(\sigma^*, \alpha^*, \hat{\mu}; h_t) \geq V(\sigma', \alpha^*, \hat{\mu}; h_t)$ for all σ' and h_t .
2. $Q(\sigma^*, \alpha^*, \hat{\mu}; h_t) \geq Q(\sigma^*, \alpha', \hat{\mu}; h_t)$ for all α' and h_t .
3. $\hat{\mu}$ evolves according to Bayes' rule⁸ using the strategies (σ^*, α^*) .

⁷I am suppressing its dependence on strategies.

⁸The full support assumption ensures that Bayes' rule is always applicable since any action by the incumbent leads to every level of voter utility with positive density.

In what follows, I will use the term *equilibrium* for perfect public equilibrium. Because I will only refer to value functions in a given equilibrium, I henceforth drop the notation emphasizing the dependence of the voter's and the politicians' value functions, V and Q , on strategy profiles. Instead, I emphasize their dependence on the history of play or incumbent reputation by writing $V(h)$ and $Q(h)$ or $V(\mu)$ and $Q(\mu)$ as appropriate.

3. Equilibrium Selection

In this section, I discuss the multiplicity of equilibria. I begin with the following example which starkly outlines the problem.

Proposition 1. *If $\lambda = 0$, any pure reelection strategy σ can be supported as part of an equilibrium.*

To see that this is true, I first identify the equilibrium with the lowest payoffs for all players in the equilibrium set, which I call an equilibrium in *grim strategies*. Suppose competent politicians always choose $a = 0$. Then, the voter is left indifferent among all politicians and may choose any reelection rule. In particular, it is a best response for him never to reelect a politician, regardless of his performance. This reelection strategy makes $a = 0$ a best response.

Next, I note that this equilibrium may be used as part of other equilibria as a credible punishment to the voter for not following a prescribed reelection strategy. Because the voter's expected payoff can never be worse than zero, the following is an equilibrium for any pure reelection strategy σ : the voter plays σ on the equilibrium path while politicians play a best response to σ . If the voter ever deviates from σ , equilibrium play switches to grim strategies.

While allowing for $\lambda > 0$ eliminates many equilibria, I will show in Section 5 that, for small λ , there is an equilibrium in which the voter's per-period expected utility is λ . This equilibrium can be used as a punishment to sustain a wide variety of voting behavior on the equilibrium path, in the same spirit as the example above.

3.1. Equilibrium Selection Criteria

I now introduce three concepts which will prove useful in narrowing our attention to a smaller set of equilibria.⁹ I will use them to evaluate candidate equilibria in Sections 4 and 5.

One may object to the equilibria of Proposition 1 by arguing that it is implausible that all politicians in P will coordinate on playing grim strategies in the continuation game. Since the physical and informational environment is identical each time a politician is elected to his first term, it seems natural to focus on equilibria in which strategies are the same every time the voter begins a fresh relationship with a politician. This, of course, implies that the value of electing a challenger for the politician is constant through all histories. I call this condition *challenger-stationarity*. Because it is sufficient for my purposes and a weaker condition, I define challenger-stationarity in terms of the value of electing a challenger rather than the continuation strategies played.

Definition 2. *An equilibrium satisfies challenger-stationarity if the value of electing a challenger is history-independent.*

Markov perfection (Maskin and Tirole (2001)) is one of the most common equilibrium refinements used in applied theory. Indeed, related work on repeated elections by Duggan (2000), Banks and Duggan (2008), and Meirowitz (2007), as well as related work on reputation games such as Mailath and Samuelson (2001) and Hörner (2002) has focused on Markov perfect equilibria. Standard arguments for Markov perfection stress the simplicity of Markovian strategies.

In the current set-up, in which only beliefs about the incumbent's type affect the set of feasible payoffs, incumbent reputation is the natural state space for

⁹A common equilibrium selection criterion which I do not emphasize is voter-optimality. Like the equilibria of Section 5, the voter optimal equilibrium involves performance cutoffs which depend on incumbent reputation. However, the equilibria are not renegotiation-proof and must be supported by trigger strategies like the equilibria in Section 4.2. Furthermore, without imposing more structure on the model, I cannot derive predictions for the dynamics of political careers in the voter-optimal equilibrium.

one to use when considering an appeal to Markov perfection. To define Markov perfection in the current setting note that different histories, possibly of different lengths, can lead to the same incumbent reputation. I can group these together to define a partition of the set of all histories as follows: if $\hat{\mu}(h^1) = \hat{\mu}(h^2)$ for $h^1, h^2 \in H$ then $h^1, h^2 \in h_M \in H_M$. H_M is the *Markovian partition* of histories.

Definition 3. *An equilibrium is Markov perfect if its strategies (σ, α) are measurable¹⁰ with respect to the Markovian partition H_M .*

In a political agency model with heterogeneous politicians, the tension between sanctioning and selection can lead to doubts about the credibility of the voter's reelection strategies. If a politician of reputation $\mu > \mu_0$ would outperform a replacement following certain histories of play, but must be fired following another, what is to stop the voter and the politician from coordinating on the mutually beneficial path of play? Can we expect voters to throw incumbents out of office even though keeping them could feasibly prove profitable? An equilibrium refinement which directly addresses this question is *weak renegotiation-proofness* (Farrell and Maskin (1989), henceforth WRP).¹¹¹²

Farrell and Maskin's definition of WRP equilibrium is as follows: an equilibrium strategy profile φ is WRP if there do not exist continuation equilibria φ^1 and φ^2 of φ such that φ^1 strictly Pareto dominates φ^2 (i.e. payoffs under φ^1 are

¹⁰Following Maskin and Tirole (2001), a strategy σ is measurable with respect to a partition of histories H_i if $h, h' \in h_i$ implies $\sigma(h) = \sigma(h')$.

¹¹Weak renegotiation-proofness is a condition of *internal consistency* that makes comparisons between the continuation payoffs of a given equilibrium strategy profile. Competing notions of renegotiation-proofness, such as that advocated by Pearce (1987), call for *external consistency* so that comparisons are made across equilibria. In particular, Pearce argues that comparisons should be made among the the lowest continuation payoffs of equilibria. Because not reelecting politicians (giving them continuation payoff of zero) is the voter's only effective tool for providing incentives, this approach is unlikely to narrow the set of equilibria in this game.

¹²Renegotiation-proofness is most often used in situations involving a small number of players where coordination seems most likely. In that sense my use of it in a voting model is non-standard. However, I argue that electoral campaigning can also be an effective coordination tool.

strictly greater for both players than under φ^2). To adapt the definition of WRP to the current game, I must take into account that the politician's reputation is payoff relevant, so that continuation payoffs when the politician's reputation is μ may not be feasible when his reputation is $\mu' \neq \mu$. Because the probability that politicians in the infinite set P will be elected to office is zero (replacements are randomly selected), regardless of the voter's decision, I write the definition of WRP in terms of the voter's and the incumbent's payoffs.

Definition 4. *An equilibrium is weakly renegotiation-proof (WRP) if, for any two histories $h^1, h^2 \in H$ leading to a reputation μ , i.e. $h^1, h^2 \in \hat{h} \in \hat{H}$, $V(h^1) > V(h^2)$ implies $Q(h^1) \leq Q(h^2)$ (and therefore $Q(h^1) > Q(h^2)$ implies $V(h^1) \leq V(h^2)$).*

Because Markovian strategies take the same (possibly mixed) action at any two histories leading to the same incumbent reputation, any Markov perfect equilibrium is WRP.

4. Sanctioning and Selection

As I discussed in the Introduction, voters must balance two goals when making reelection decisions. They must give incumbents incentives to exert effort while attempting to keep incumbents who have shown themselves to be "good types" or, in the context of this model, competent. While the dual role of elections is well-understood, sanctioning and selection are not clearly defined game-theoretically.¹³ In this section, I propose a definition based on the information used by voters to

¹³A notable exception is Ashworth, Bueno de Mesquita, and Friedenberg (2010) who argue that, even in a two-period model in which the voter must reelect an incumbent who is more likely to be a good type, sanctioning takes place in the form of ex-ante standard setting by the voter. The standards chosen by the voter have an effect on equilibrium effort and, thus, on the voter's own inference about types. Seen in this light, sanctioning is defined as equilibrium selection motivated by the voter's power to coordinate expectations. In contrast, I define sanctioning and selection in terms of reelection strategies.

make reelection decisions. I then consider the possibility of having selection-only and sanctioning-only equilibria.

Definition 5. *The the partition of histories generated by the reelection strategy σ , H_σ , is the coarsest partition of histories with respect to which σ is measurable.*

A partition of histories H_j is σ -consistent if H_σ is a refinement of H_j or vice-versa.

Intuitively, an equilibrium involving selection is one in which the voter bases his reelection decisions on his beliefs about the incumbent's type. Such a strategy must use a subset of the information contained in the Markovian partition in Section 3.1. Similarly, in an equilibrium with sanctioning the voter makes reelection decisions based on the incumbent's most recent performance.¹⁴ In this case, the voter's strategy must be measurable with respect to the *retrospective partition* of histories in which any two histories ending in the same incumbent performance r are grouped together: $h^1, h^2 \in h_R \in H_R$ whenever $r(h^1) = r(h^2)$.

Definition 6. *An equilibrium exhibits sanctioning if the retrospective partition of histories H_R is σ -consistent.*

An equilibrium exhibits selection if the Markovian partition of histories H_M is σ -consistent.

An equilibrium is sanctioning-only (selection-only) if H_σ is a coarsening of H_R (H_M).

This typology is related to the classification of voting into retrospective and prospective. In retrospective voting, reelection decisions are based on past performance – possibly coinciding with my definition of sanctioning. In prospective voting, the voter considers information relevant the incumbent's expected future performance – possibly coinciding with my definition of selection. However, the similarities are only skin-deep. A Markovian reelection strategy is retrospective

¹⁴This definition could be generalized to include the previous n realizations of voter utility.

because it considers past performance through reputation. However, it does not exhibit sanctioning. Indeed, any equilibrium of the repeated game will necessarily involve both retrospective and prospective voting: information on past performance is used to evaluate incumbents, yet a reelection strategy is only a best response if expected future performance is maximized.

4.1. Selection-Only Equilibria

Selection-only equilibria are Markov perfect. Their existence is easily verified when $\lambda = 0$ as equilibrium in grim strategies, discussed above, provides a trivial example of a Markov perfect equilibrium. When λ is large enough, the voter's problem approximates a pure selection problem and equilibria in which the voter uses a reputation cut-off reelection rule exist. However, it is not clear that Markov perfect equilibria exist when λ is positive and small. Even if they do, the following result shows that Markovian strategies do not enable the voter to overcome the moral hazard problem.

As mentioned in Section 3.1, Markov perfection is commonly used in the literature on repeated elections. Furthermore, many papers use two period models in which reelection is based only on the voter's beliefs about the incumbent's type¹⁵. Banks and Sundaram (1993, p. 310) end their article by asking whether 'interesting' equilibria which are stationary in reputation exist. Theorem 1 answers in the negative, at least for this slightly simpler setting. Note that the result implies that, for $\lambda = 0$, no politician will ever exert positive effort in a Markov perfect equilibrium.

Theorem 1. *There exists a $\bar{\lambda}_M > 0$ such that, for all $\lambda \in [0, \bar{\lambda}_M)$,¹⁶ the voter's payoffs in every Markov perfect equilibrium are bounded above by the zero-effort*

¹⁵See Reed (1994), Banks and Sundaram (1998), Fearon (1999), Berganza (2000), and Ashworth (2005). In these models with term limits, it is assumed that high types perform better than low types in the last term in which no incentives for effort can be provided. Therefore, incumbents are reelected if their expected type is higher than that of a replacement.

¹⁶I conjecture, but have not been able to prove, that this result holds for all λ .

utility of having a competent incumbent ($V(\mu_0) \leq \lambda$).

A full proof is provided in the Appendix (Section 8.2). To develop some of the intuition behind the proof, suppose that politicians of all reputations provide effort of at least $\hat{a} > 0$ in equilibrium. If reelection strategies depend only on reputation, the politician's ex-ante value of acquiring a reputation μ is $\sigma(\mu)Q(\mu) = \hat{Q}(\mu)$. Because posterior reputation is increasing in performance, in order to provide incentives for effort the function $\hat{Q}(\mu)$ must be increasing in reputation. As a politician's reputation nears 1, the change in his reputation for a fixed but wide set of outcomes (r) approaches zero. Therefore, the politician's value function \hat{Q} must increase at least a fixed amount (itself dependent on \hat{a}) in each of an infinity of ever smaller intervals. However, I know that \hat{Q} is bounded above by the value of holding office forever while exerting zero effort: $\frac{u(0)}{1-\delta}$. Therefore, providing incentives for effort at least \hat{a} for all reputations is infeasible. Conversely, if politicians of reputation at least μ do not provide effort, it is not worthwhile for the voter to reelect them. This in turn, means that politicians should avoid ending up with a reputation higher than μ , and they can only do this by providing lower effort, leading to an unraveling of incentives for incumbents of all reputations.

The result, and its proof, echo Proposition 2 in Mailath and Samuelson (2001) (henceforth MS). There are, however, important differences. For instance, the relation between prices and reputation assumed in MS gives the agent built-in incentives to improve his reputation which are absent in the current setting. Interestingly, the presence of continuous noise (and, thus, continuous outcomes) in my model rules out the type of partition of reputation space which makes mixed strategy equilibria with positive effort possible in MS.

4.2. Sanctioning-Only Equilibria

If focusing only on selection leads to low payoffs for voters, one might wonder if focusing only on sanctioning instead is a feasible way for voters to obtain more favorable outcomes. In a closely related paper, Banks and Sundaram (1993) de-

scribe equilibria of the repeated elections game¹⁷ in which voters use performance standards to incentivize incumbents to exert effort, and in which reelection strategies do not vary with incumbent reputation. In these equilibria, politicians are held to a fixed performance standard r^* . When this performance standard is not met, the politician is not reelected.

Because incumbents face a constant standard for reelection, their effort choices are constant through time. To see this, consider a competent incumbent's problem:

$$\max\{u(a) + \delta(1 - F(r^*|a))Q\}$$

Note that I write the value of being in office as a constant Q because, given this reelection strategy, the history of play will not have any effect on future play. Clearly, the solution to this problem is some constant level of effort a^* which leads to a constant probability of reelection $1 - F(r^*|a^*)$. Thus, these equilibria predict no career dynamics of interest.

As an incumbent's reputation increases, so does his expected future performance: μa^* per-period. This raises the question of why voters are willing to dismiss incumbents who have developed good reputations, given that they would normally outperform a randomly-drawn challenger. The answer is that this strategy is enforced through the following trigger strategy: after a politician has missed his performance target once, he never expects to be reelected again and will therefore never again exert effort. The equilibrium construction is akin to that used in Proposition 1.

A serious criticism of these equilibria is that after a politician with high reputation misses a performance target, both the voter and the politician would benefit from agreeing to keep the politician in office and continue play as if the incumbent had not violated the voter's performance standard. Therefore, the punishment prescribed by the equilibrium is not credible. That is, the equilibria are not WRP.

¹⁷The model in Banks and Sundaram (1993) includes an arbitrary but finite number of politician types who differ in their cost of providing effort. This paper's model is a special case with low types having arbitrarily high cost of effort.

5. Equilibria in Reputation-Dependent Performance Cutoffs (RDC)

In this section, I define the class of RDC equilibria and prove their existence. These equilibria exhibit both sanctioning and selection, they satisfy challenger-stationarity, and they are WRP. Furthermore, they allow the voter to overcome the moral hazard problem and enjoy per-period expected utility greater than λ . The equilibria are a generalization the equilibria of Ferejohn (1986) to the current setting. Ferejohn (1986) builds his equilibria on two basic insights:

1. Performance cutoffs are effective means of providing incentives to politicians.
2. If voters are indifferent between electing incumbents and replacements, they can credibly commit to using performance cutoff reelection strategies.

Voter indifference provides a solid foundation on which to construct equilibria. Its relation to WRP is the subject of Section 6.

Definition 7. *An equilibrium satisfies voter indifference if $V(\mu) = V(\mu_0)$ whenever $\sigma(h) > 0$ for some history satisfying $\hat{\mu}(h) = \mu$.*

Intuitively, voter indifference can be seen as a formalization of the often-voiced sentiment: "One politician is as bad as another." This does not mean that there are no differences among politicians, but rather that they all exploit the system in their favor to the point where there are no differences in expected performance.

If the voter is indifferent between incumbents and replacements, it must be that $V(\mu) = V(\mu_0) = V$. Therefore, the value to having any incumbent in office is V and the voter's value function may be written as:

$$V = \mu(\lambda + \alpha(\mu)) + \delta V \tag{5.1}$$

Solving for $\alpha(\mu)$, we find that: $\alpha(\mu) = \frac{V(1-\delta)}{\mu} - \lambda$. Denoting $v = V(1 - \delta)$, I write the identity for effort levels which keep the voter indifferent among politicians as:

$$\alpha(\mu) = \frac{v}{\mu} - \lambda \tag{5.2}$$

I refer to v as the *value to the voter* of an effort profile $\alpha(\mu)$. Note that $\alpha'(\mu) = -\frac{v}{\mu^2}$ so that effort is decreasing in reputation. Clearly, any equilibrium in which $v \geq \lambda$ will involve a lowest reputation politician which will ever be elected because $\alpha(\mu) \rightarrow \infty$ as $\mu \rightarrow 0$.

Definition 8. *An equilibrium in reputation-dependent performance cutoffs (RDC) with value v is an equilibrium in which:*

- *Politicians follow an effort strategy $\alpha(\mu) = \frac{v}{\mu} - \lambda$.*
- *The voter follows a reputation-dependent performance threshold reelection strategy: $\sigma(r_t, \mu_{t-1}) = \begin{cases} 1 & \text{if } r_t \geq r(\mu_{t-1}) \\ 0 & \text{otherwise} \end{cases}$*

As a measure of their simplicity, such strategies are Markovian if we take the pair (μ_{t-1}, μ_t) rather than μ_t as the state variable. This condition is equivalent to constraining strategies to depend only on reputation and performance: (r_t, μ_t) or (μ_{t-1}, r_t) .

Because effort strategies $\alpha(\mu)$ keep the voter indifferent among reelection strategies, if there exists a performance cutoff function $r(\mu) : [0, 1] \rightarrow \mathbb{R}$ which makes $\alpha(\mu)$ a best response, this will be an equilibrium. The following Theorem states the existence of equilibria in reputation-dependent cutoff strategies for low values of λ .

Theorem 2. *There exist $\bar{\lambda}$ and \bar{v} , $\bar{v} > \bar{\lambda} > 0$, such that for all $\lambda \in [0, \bar{\lambda})$ and any $v \in [\lambda, \bar{v}]$, there is an equilibrium in reputation-dependent performance cutoffs (RDC) with value v .*

The proof (in Section 8.1) proceeds as follows: let $Q(\mu)$ be any bounded and well-behaved candidate for the politician's value function. If λ is small and I have

chosen v carefully, it will be obtainable under $Q(\mu)$ in an RDC equilibrium since $Q(\mu)$ is bounded below by $u(0)$, making it worthwhile for an incumbent to exert some effort in order to make his reelection more likely. I then define an operator $T^v(Q)(\mu) = u(\alpha(\mu)) + \delta \int_{r_Q(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f(r|\alpha(\mu)) dr$ where $r_Q(\mu)$ is a reputation-dependent performance cutoff implementing v . A fixed point of T^v will be a value function Q with associated cutoff function $r_Q(\mu)$ implementing an effort strategy $\alpha(\mu) = \frac{v}{\mu} - \lambda$. Because this effort strategy leaves the voter indifferent between reelecting the incumbent or not, the cutoff function describes a reelection strategy which is a best response. Therefore, once I check sufficient conditions for a fixed point of T^v , I have found an RDC equilibrium. If λ is too large, the minimum level of effort required of first-term incumbents in order to satisfy voter indifference, $\lambda \frac{1-\mu_0}{\mu_0}$, is too high to be implementable.

5.1. Career Dynamics and Comparative Statics

In this section, I describe some properties of RDC equilibria. I begin with a definition:

Definition 9. *Let $\hat{t}(t)$ be the date at which the date- t incumbent was first elected to office. An incumbent's tenure at time t is the total number of periods he has served in office: $\tau(t) = t - \hat{t}(t)$.*

In any RDC equilibrium, expected reputation increases with tenure. To see this, consider the following argument. Because, in equilibrium, the voter correctly anticipates the incumbent's behavior as a function of his type, the expected reputation of the incumbent after serving a term is the same as his reputation at the beginning of the term. In other words, a given incumbent's reputation is a martingale: $E_t(\mu_{i,t+1}) = \mu_{i,t}$. However, those incumbents who end the term with the lowest reputation will be thrown out of office, leading us to conclude that expected reputation will increase every time an incumbent is reelected. The following Proposition holds for all RDC equilibria, except the case in which $\lambda = v = 0$.

Proposition 2. *In any RDC equilibrium with value $v > 0$:*

1. *An incumbent's expected level of reputation conditional on tenure is strictly increasing in tenure: $E(\mu|\tau) > E(\mu|\tau')$ whenever $\tau > \tau'$.*
2. *Politicians will fully reveal their type if they can stay in office forever: $\lim_{\tau \rightarrow \infty} E(\mu|\tau, CP) = 1$ and $\lim_{\tau \rightarrow \infty} E(\mu|\tau, IP) = 0$.*
3. *However, any incumbent will be thrown out of office in finite time: $\lim_{t \rightarrow \infty} \lim_{K \rightarrow \infty} \Pr(\tau(t) \geq K) = 0$.*

Proof. To prove part 1, consider the expected reputation (μ') of an incumbent with initial reputation μ conditional on meeting a performance standard $r(\mu)$:

$$\begin{aligned}
\mu' &= \int_{r(\mu)}^{\infty} \hat{\mu}(r, \mu) \frac{(\mu f(r|\alpha(\mu)) + (1 - \mu) f_{IP}(r))}{1 - \mu F(r(\mu)|\alpha(\mu)) - (1 - \mu) F_{IP}(r(\mu))} dr \\
&= \int_{r(\mu)}^{\infty} \frac{\mu f(r|\alpha(\mu))}{(\mu f(r|\alpha(\mu)) + (1 - \mu) f_{IP}(r))} \frac{(\mu f(r|\alpha(\mu)) + (1 - \mu) f_{IP}(r))}{1 - \mu F(r(\mu)|\alpha(\mu)) - (1 - \mu) F_{IP}(r(\mu))} dr \\
&= \frac{\mu}{1 - \mu F(r(\mu)|\alpha(\mu)) - (1 - \mu) F_{IP}(r(\mu))} \int_{r(\mu)}^{\infty} f(r|\alpha(\mu)) dr \\
&= \frac{\mu (1 - F(r(\mu)|\alpha(\mu)))}{1 - \mu F(r(\mu)|\alpha(\mu)) - (1 - \mu) F_{IP}(r(\mu))}
\end{aligned}$$

The monotone likelihood ratio assumption implies that $F(r(\mu)|\alpha(\mu)) < F_{IP}(r(\mu))$ so that $\mu' > \mu$ whenever $r(\mu)$ is finite.

To see that incumbents fully reveal their type if they can stay in office forever, recall that in an RDC equilibrium with value $v > 0$, any CP's expected performance is weakly greater than v each period. Consider the following test: the voter calculates a sample average of an incumbent's performances $\bar{r}_n = \frac{1}{n} \sum_{t=1}^n r_t$. For all n , $E(\bar{r}_n|CP) \geq v$. Also, $Var(\bar{r}_n) = \frac{1}{n^2} n\theta = \frac{\theta}{n}$ where θ is the variance of ε . Thus, $\lim_{n \rightarrow \infty} Var(\bar{r}_n) = 0$. Therefore, the probability that $\bar{r}_n < v$ goes to zero as the number of observations (n) grows arbitrarily large. In this way, the voter may

perfectly infer an incumbent's type, given enough observations of his performance in office.

To see that any incumbent is thrown out of office in finite time with probability one, observe first that CPs always survive with a higher probability than IPs. This is the case because CPs exert effort so that the distribution of outcomes when they are in office first order stochastically dominates that of IPs. Because the effort level required by RDC equilibrium grows unboundedly as reputation approaches zero, there is a lowest reputation level which is ever reelected with positive probability. The full support assumption ensures that this region of no-reelection is reached with positive probability $p(\mu) > 0$ from any initial reputation μ . Therefore, reelection probabilities are bounded above by $1 - p(\mu)$. Because each of these bounds is less than one, the probability of remaining in office at least n periods goes to zero as n grows unboundedly: $\lim_{n \rightarrow \infty} (1 - p(\mu_n))^n = 0$ for any series of reputations μ_n . ■

A straight-forward implication of RDC equilibrium is that effort decreases with reputation. This, along with Claim 2, implies a negative relationship between expected performance and tenure for a given politician (though not across politicians).

Proposition 3. *For a given politician, expected effort and performance are negatively related to tenure.*

Proof. Our variables of interest are $E(r_t|\tau(t), i)$ and $E(a_t|\tau(t), i)$. A given politician i is either competent or incompetent. If he is incompetent, then his expected effort and performance are zero, regardless of his tenure: $E(r_t|\tau(t), i) = 0 \forall \tau$. If the politician is competent $E(r_t|\tau(t), i) = E(\lambda + a_t + \varepsilon_t|\tau(t), i) = E(\lambda + a_t|\tau(t), i)$.

If i is competent, then his expected effort and performance will be decreasing in his reputation: $\frac{\partial a_t}{\partial \mu_t} = -\frac{v}{\mu_t^2}$. By Claim 2, on average, reputation is increasing in tenure. Thus, i 's expected effort and performance will be decreasing in his tenure.

■

This is a prediction which has been emphasized by others, including Banks and Sundaram (1998) and Ashworth (2005), though their derivation relies on last-period effects. As Ashworth (2005) points out, the prediction fits well with the negative correlation between tenure and personal constituent services in the U.S. House of Representatives examined in Cain, Ferejohn and Fiorina (1990). In a study of the U.S. senate, Levitt (1996) finds some evidence of a positive correlation between ideological shirking and tenure.

Galasso, et al. (2009) find a negative relationship between tenure and attendance in the Italian legislature. Attendance may be interpreted as an observation of effort in this context if I reinterpret the model to fit Italy's parliamentary system. In this case, politicians are directly accountable to their party rather than to the voters. One might imagine that parties face a similar retention problem to that faced by voters in democracies with direct representation, and thus may use RDC strategies.

An important question we may ask is whether heterogeneity among politicians is good or bad for voters. Clearly, if there are only incompetent politicians, the voter will have expected utility of zero. By Theorem 2 the voter can achieve strictly positive expected utility in an RDC equilibrium whenever $\mu_0 > 0$, so that moving from a state of the world with only incompetent politicians to one in which some may be competent is good for the voter. A world in which all politicians are competent takes us back to the environment of Ferejohn 1986 in which voters can achieve positive utility. The following proposition shows that adding the possibility of incompetent politicians is never good for voters.

Proposition 4. *The highest expected payoff to the voter in an RDC equilibrium is weakly increasing in the proportion of high types (μ_0) in the set of politicians P , and is strictly increasing for some μ_0 .*

Proof. Given an RDC equilibrium under μ_0 , the same strategies may be used when new politicians are more likely to be a CP ($\mu'_0 > \mu_0$). This implies that the highest achievable voter utility is weakly increasing in μ_0 . However, we know that

as $\mu_0 \rightarrow 0$, so do the feasible payoff levels for voters since $\alpha(\mu_0) \rightarrow \infty$. Therefore, for any RDC equilibrium with positive value to the voter, there is a μ_0 at which this value is not feasible so that there is a μ_0 at which the highest expected payoff to the voter is strictly increasing. ■

6. Renegotiation-Proofness and Voter Indifference

Voter indifference (Definition 7) plays a central role in both RDC equilibrium, and in the equilibria of standard political agency models based on Ferejohn (1986). Thus, providing some theoretical justification for its use is important. Voter indifference implies WRP since continuation payoffs are the same for the voter after any history of play, ruling out Pareto improvements. The reverse implication, that WRP implies voter indifference, can only be partly proven. Specifically, in any WRP and challenger stationary (Definition 2) equilibrium, the indifference condition will hold for a set of reputations of positive measure. Outside of this set, reelection strategies are pure and Markovian.

Theorem 3. (1) *Any equilibrium satisfying voter indifference is weakly renegotiation-proof (WRP).*

(2) *If $\lambda < \bar{\lambda}_M$,¹⁸ in any equilibrium satisfying weak renegotiation-proofness (WRP), $V(\mu_0) > \lambda$, and challenger-stationarity the following conditions hold:*

- *Every reputation interval $[\mu, 1]$ contains a subset S of strictly positive measure such that, for any $m \in S$, if $\hat{\mu}(h) = m$ then $V(h) = V(\mu_0)$.*
- *For any $m \in S^C = [\mu, 1] \setminus S$, if $h^1, h^2 \subset h_M(m)$ then, either $\sigma(h^1) = \sigma(h^2) = 1$ or $\sigma(h^1) = \sigma(h^2) = 0$. That is, strategies in the complement of S are Markovian and degenerate.*

Proof. Consider any reputation m such that one can find histories h^1 and h^2 satisfying $\hat{\mu}(h^1) = \hat{\mu}(h^2) = m$, $\sigma(h^1) = 1$ and $\sigma(h^2) = 0$ (or strategies are mixed

¹⁸Recall that $\bar{\lambda}_M$ is the upper bound on λ needed for Theorem 1 to hold.

but may lead to reelection after h^1 and dismissal after h^2). Then WRP implies that, because $Q(h^1) > Q(h^2)$, $V(h^1) \leq V(h^2)$. Also, because it is a best response to reelect after h^1 , $V(h^1) \geq V(\mu_0)$. Because it is a best response not to reelect after h^2 , $V(h^2) = V(\mu_0)$. From this I conclude that $V(h^1) = V(h^2) = V(\mu_0)$.

This leaves reputation levels at which incumbents are always reelected or always thrown out of office. However, any reelection strategy leading to this sort of behavior over almost all reputations is an essentially Markovian reelection strategy. By the generalization of Theorem 1 in the appendix, this contradicts the premise that $V(\mu_0) > \lambda$. ■

As it relates to the model of Ferejohn (1986), the relationship between WRP and voter indifference solidifies the foundations of equilibria in performance cut-offs. Even if one allows for heterogeneity among politicians, there is an intuitively appealing equilibrium refinement (WRP) which leads back to voter indifference. Thus, its use as a commitment device is both credible and focal.

7. Conclusions

The aim of this paper has been to improve the general understanding of the dual role of elections: selecting competent politicians and incentivizing them to exert costly effort. In particular, I have focused on the interaction between a politician's reputation, the voter's willingness to replace him with a lesser known candidate, and the politician's performance. I have done so in the context of a simple model of repeated elections without term limits which does not impose the desirability of competence even in the absence of incentivizing mechanisms.

As in many infinitely repeated games, the problem of equilibrium selection takes center stage. However, attention paid to this issue has been rewarded in unexpected ways. I have shed light on the question of whether voters can effectively incentivize politicians by simply conditioning reelection on reputation. The answer is no (Theorem 1), at least in the setting I study. I have uncovered an interesting relationship between weak renegotiation-proofness and the condition

that the voter be left indifferent among politicians of different reputations and, therefore, between reelecting an incumbent and electing a challenger (Theorem 3). This has given us fresh perspective on a seminal work in political agency, Ferejohn (1986), and increased confidence in its underlying logic. Finally, I have considered some of the virtues and limitations of the large set of equilibria in trigger strategies.

My exploration of the equilibrium set and its refinements led me to generalize the equilibria of Ferejohn (1986) to a model with heterogeneous politicians (RDC equilibrium, Section 5). The use of voter indifference to support performance cutoffs which, in turn, allow the voter to incentivize effort from politicians is consistent with several intuitively appealing equilibrium refinements. Additionally, after establishing existence (Theorem 2), I go on to explore the predictions of the model for political careers. The results presented in Section 5.1 replicate those derived in similar models with term limits and in which incumbent type directly affects voter utility. That they continue to hold when there are no term limits and politicians differ only in competence should encourage researchers to look for evidence of these career dynamics in contexts such as the U.S. Congress and understand them as a consequence of political agency.

I conclude with some thoughts on the direction of future research. The theoretical literature has provided predictions of the relation between different institutional assumptions and different ways of modeling politician heterogeneity, and predictions about voter behavior and political careers. This variation in predictions should be exploited in empirical work to verify the nature of the differences among politicians. Bobonis, Cámara Fuertes, and Schwabe (2011) present a first attempt at doing this. Other promising lines of research include endogenizing politician entry to develop an understanding of the causes of politician heterogeneity, and identify policies to encourage the entry of competent and honest citizens into politics.

8. Appendix

For easy reference in the proofs that follow, I rewrite and label the assumptions on the density function $f(r|a)$ discussed in Section 2.1.

$$\text{Full support: } f(r|a) > 0 \text{ for all } r \text{ and } a. \quad (\text{A1.})$$

$$f(r|a) \text{ is twice continuously differentiable in both arguments.} \quad (\text{A2.})$$

$$\text{Monotone Likelihood Ratio Property (MLRP): } \frac{f(x|a)}{f(x|a')} > \frac{f(y|a)}{f(y|a')} \text{ whenever } x > y \text{ and } a > a'. \quad (\text{A3.})$$

$$\text{Symmetry: } f(r|a) \text{ is symmetric around its mean.} \quad (\text{A4.})$$

$$\text{Strict Concavity: } -u''(a) > \max_Q \delta \int f_{aa}(r|a)Q(r)dr \text{ for } Q : \mathbb{R} \rightarrow [u(0), \frac{u(0)}{1-\delta}]. \quad (\text{A5.})$$

The strict concavity assumption ensures that the CP's optimal effort choice can be characterized by the first order condition to his maximization problem. If $f(r|a)$ is the density of the normal distribution with mean a and variance 1, $-u''(a) > \frac{1}{2} \frac{\delta}{1-\delta} u(0)$ for all a is a sufficient condition.

It will be useful in the proof of Lemma 1 to note that A3. implies that $\frac{\partial^2 \ln f(r|a)}{\partial r \partial a} > 0$.

A straightforward but useful implication of assuming the independence of noise from effort levels is that $f(r|a) = f(r - a|0)$. Another way of stating this is:

$$f(r|a) = f(r + k|a + k) \text{ for any } k \in \mathbb{R}. \quad (8.1)$$

In what follows, $f_a(r|a) = \frac{\partial f(r|a)}{\partial a}$, $f_{aa}(r|a) = \frac{\partial^2 f(r|a)}{\partial a^2}$ and $\hat{\mu}_2(r, \mu) = \frac{\partial \hat{\mu}(r, \mu)}{\partial \mu}$.

8.1. Existence of RDC equilibria - proof of Theorem 2

I proceed by determining reputation-dependent performance cutoffs which implement effort levels which make the voter's expected utility constant across reputations. Once I have done this, I define an operator which, for any well-behaved candidate value function for the incumbent, determines performance cutoffs and a new candidate value function. A fixed point of this operator gives us an incumbent value function and associated performance cutoffs. Because, at every reputation, the voter is indifferent between reelecting the incumbent and electing a challenger, using these performance cutoffs as a reelection strategy is sequentially rational for the voter. Thus, the following four elements describe an equilibrium: value functions for the politician and the voter, effort strategies which keep the voter's value function constant, and reelection strategies which use the derived performance cutoffs to make reelection decisions. In order to guarantee the existence of a fixed point, I must check that the conditions for Schauder's fixed point theorem hold. I do so in a series of Lemmas.

When facing a reputation-dependent performance cutoff, a CP with reputation μ solves the problem:

$$\max_a \left\{ u(a) + \delta \int_{r(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f(r|a) dr \right\} \quad (8.2)$$

To implement performance v (or effort strategy $\alpha(\mu) = \frac{v}{\mu} - \lambda$) with a reputation-dependent cutoff $r(\mu)$ the politician's first order condition (FOC) must be satisfied at $\alpha(\mu)$:

$$u'(\alpha(\mu)) + \delta \int_{r(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f_a(r|\alpha(\mu)) dr = 0 \quad (8.3)$$

The FOC uniquely determines the incumbent's action since, by Strict Concavity (A5.),

$$u''(\alpha(\mu)) + \delta \int_{r(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f_{aa}(r|\alpha(\mu)) dr < 0 \quad (8.4)$$

The FOC must hold at every reputation point μ so that the derivative of the FOC with respect to μ must be 0:

$$u''(\alpha(\mu))\alpha'(\mu) - \delta r'(\mu) Q(\hat{\mu}(r(\mu), \mu)) f_a(r(\mu)|\alpha(\mu)) + \delta \int_{r(\mu)}^{\infty} Q'(\hat{\mu}(r, \mu)) \hat{\mu}_2(r, \mu) f_a(r|\alpha(\mu)) + Q(\hat{\mu}(r, \mu)) f_{aa}(r|\alpha(\mu)) \alpha'(\mu) dr = 0$$

Solving for $r'(\mu)$:

$$r'(\mu) = \frac{\frac{1}{\delta} u''(\alpha(\mu))\alpha'(\mu) + \int_{r(\mu)}^{\infty} Q'(\hat{\mu}(r, \mu)) \hat{\mu}_2(r, \mu) f_a(r|\alpha(\mu)) + Q(\hat{\mu}(r, \mu)) f_{aa}(r|\alpha(\mu)) \alpha'(\mu) dr}{f_a(r(\mu)|\alpha(\mu)) Q(\hat{\mu}(r(\mu), \mu))} \quad (8.5)$$

The Fundamental Theorem of Differential Equations guarantees the existence of a function $r(\mu)$ satisfying the equation above as long as the first order condition is feasible and I can bound $r(\mu)$ away from the point where $f_a(r(\mu)|\alpha(\mu)) = 0$ (for symmetric distributions, this point is $\alpha(\mu)$), since the RHS of the expression above is continuous and the domain of $r(\mu)$ is compact.

Before presenting a proof of existence of these equilibria, I select a feasible value for the voter: $v > \lambda$. For analytical convenience, I focus on cutoffs where $r(\mu) - \alpha(\mu) > \lambda$ and $f_a(r(\mu)|\alpha(\mu)) > 0$.

A lower bound for the value of holding office is $\bar{Q} = u(0)$. To emphasize its dependence on v , I write $\alpha(\mu, v) = \frac{v}{\mu} - \lambda$ for the incumbent's effort strategy. Using this lower bound as a hypothetical constant value function and invoking condition 8.1:

$$-u'(\alpha(\mu, v)) = \delta \int_{r(\mu)}^{\infty} \bar{Q} f_a(r|\alpha(\mu, v)) dr = \delta \bar{Q} f(r(\mu)|\alpha(\mu, v)) \quad (8.6)$$

By Strict Concavity (A5.), $-u'(\alpha(\mu, v))$ grows unboundedly. Therefore, this

equality cannot hold for v large enough. However, as v and λ jointly approach zero, $\alpha(\mu, v) \rightarrow 0$ and therefore $u'(\alpha(\mu, v)) \rightarrow 0$. However, $\bar{Q} > 0$, so that the equation must hold for appropriate $r(\mu)$ for v low enough (but still strictly positive). Indeed, for small enough λ , I can guarantee that a $v > \lambda$ may be sustained as above even if I restrict attention to cutoffs satisfying $r(\mu) - \alpha(\mu) > B$ for any given lower bound B . This will be useful when proving Lemma 1. Let $\bar{v}_1 > \bar{\lambda}_1 > 0$ be upper bounds on v and λ which satisfy these requirements.

I now present the fixed point problem, referring to the derivations above as they become useful.

Definition 10. $C([\mu_0, 1])$ is the space of bounded, continuous functions $f : [\mu_0, 1] \rightarrow \mathbb{R}$.

$\hat{C} \subset C([\mu_0, 1])$ is the restriction of this space to functions with K -bounded first derivative and codomain $[u(0), \frac{u(0)}{1-\delta}]$.

It is clear that \hat{C} is non-empty, bounded, closed, and convex.

Definition 11. Given a function $Q \in \hat{C}$ and a value $v \geq \lambda$, let $r_Q(\mu)$ be a function satisfying Equation 8.5.

Definition 12. The operator $T^v : \hat{C} \rightarrow \hat{C}$ is:

$$T^v(Q)(\mu) = u(\alpha(\mu, v)) + \delta \int_{r_Q(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f(r|\alpha(\mu, v)) dr$$

A fixed point of this operator will define a value function for the politician in a reputation-dependent cutoff equilibrium. To prove the existence of a fixed point, I will use Schauder's fixed point theorem. Schauder's theorem is a generalization of Brouwer's fixed point theorem to infinite-dimensional spaces. For a proof, see Lusternik and Sobolev (1974).

Theorem 1 (Schauder's Fixed Point Theorem). Let X be a bounded subset of \mathbb{R}^m , and let $C(X)$ be the space of bounded continuous functions on X , with the sup norm. Let $F \subset C(X)$ be nonempty, closed, bounded and convex. If the mapping $T : F \rightarrow F$ is continuous and the family $T(F)$ is equicontinuous, then T has a fixed point in F .

I must first verify that T^v maps \hat{C} to \hat{C} .

That $T^v(Q)$ is continuously differentiable in μ is immediate from the differentiability of f , Q , $\alpha(\mu)$, and $r_Q(\mu)$.

That $T^v(Q)$ has a K -bounded derivative is verified in the following Lemma. It will be useful in proving the Lemma to note that the first derivative with respect to μ of the Bayesian updating function is:

$$\frac{\partial \hat{\mu}(r, \mu)}{\partial \mu} = \hat{\mu}_2(r, \mu) = \frac{f(r|\alpha(\mu))f_{IP}(r) + \mu(1 - \mu)\alpha'(\mu)f_a(r|\alpha(\mu))f_{IP}(r)}{(\mu f(r|\alpha(\mu)) + (1 - \mu)f_{IP}(r))^2} \quad (8.7)$$

Note that $\lim_{\lambda \rightarrow 0} \lim_{v \rightarrow 0} \hat{\mu}_2(r, \mu) \rightarrow 1$. The second term in the numerator converges to zero since $f_a(r|\alpha(\mu))f_{IP}(r)$ is uniformly bounded above and $\alpha'(\mu) = -\frac{v}{\mu^2} \rightarrow 0$.

Lemma 1. *There exist $\bar{v}_2 > \bar{\lambda}_2 > 0$ such that, for any $\lambda < \bar{\lambda}_2$ and $v < \bar{v}_2$, $\left| \frac{\partial T^v(Q)}{\partial \mu} \right| < K$ for any $\mu \in [\mu_0, 1]$ and any continuously differentiable function Q with absolutely K -bounded first derivative.*

Proof. $\frac{\partial T^v(Q)}{\partial \mu} = \frac{\partial [u(\alpha(\mu)) + \delta \int_{r(\mu)}^{\infty} Q(\hat{\mu}(r, \mu))f(r|\alpha(\mu))dr]}{\partial \mu} =$

$$\begin{aligned} & u'(\alpha(\mu))\alpha'(\mu) + \delta \int_{r(\mu)}^{\infty} \alpha'(\mu)Q(\hat{\mu}(r, \mu))f_a(r|\alpha(\mu))dr \\ & + \delta \int_{r(\mu)}^{\infty} Q'(\hat{\mu}(r, \mu))\hat{\mu}_2(r, \mu)f(r|\alpha(\mu))dr - \delta r'(\mu)Q(\hat{\mu}(r(\mu), \mu))f(r(\mu)|\alpha(\mu)) \end{aligned}$$

The first two terms add up to zero by the politician's FOC. Substituting Equation 8.5 into the fourth term:

$$\begin{aligned} & \delta \int_{r(\mu)}^{\infty} Q'(\hat{\mu}(r, \mu))\hat{\mu}_2(r, \mu)f(r|\alpha(\mu))dr - \frac{f(r(\mu)|\alpha(\mu))}{f_a(r(\mu)|\alpha(\mu))}u''(\alpha(\mu))\alpha'(\mu) \\ & - \frac{f(r(\mu)|\alpha(\mu))}{f_a(r(\mu)|\alpha(\mu))}\delta \int_{r(\mu)}^{\infty} Q'(\hat{\mu}(r, \mu))\hat{\mu}_2(r, \mu)f_a(r|\alpha(\mu)) + Q(\hat{\mu}(r, \mu))f_{aa}(r|\alpha(\mu))\alpha'(\mu)dr. \end{aligned}$$

I first consider the terms which include Q' . Combining them gives:

$$\begin{aligned} & \left| \int_{r(\mu)}^{\infty} \hat{\mu}_2(r, \mu)Q'(\hat{\mu}(r, \mu)) \left(f(r|\alpha(\mu)) - \frac{f(r(\mu)|\alpha(\mu))}{f_a(r(\mu)|\alpha(\mu))}f_a(r|\alpha(\mu)) \right) dr \right| \\ & < K \left| \int_{r(\mu)}^{\infty} \hat{\mu}_2(r, \mu) \left(f(r|\alpha(\mu)) - \frac{f(r(\mu)|\alpha(\mu))}{f_a(r(\mu)|\alpha(\mu))}f_a(r|\alpha(\mu)) \right) dr \right| \\ & < K \left| \frac{f(r(\mu)|\alpha(\mu))}{f_a(r(\mu)|\alpha(\mu))} \int_{r(\mu)}^{\infty} \hat{\mu}_2(r, \mu)f_a(r|\alpha(\mu))dr \right| \end{aligned}$$

Where I use the Monotone Likelihood Ratio Property (A3.) to derive both inequalities as it guarantees that the terms involving $f(r|\alpha(\mu))$ and $f_a(r|\alpha(\mu))$ will

not change sign. To see this, note that, for $f(r|\alpha(\mu)) - \frac{f(r(\mu)|\alpha(\mu))}{f_a(r(\mu)|\alpha(\mu))} f_a(r|\alpha(\mu)) \geq 0$ we must have $1 \geq \frac{f(r(\mu)|\alpha(\mu))}{f_a(r(\mu)|\alpha(\mu))} \frac{f_a(r|\alpha(\mu))}{f(r|\alpha(\mu))}$. The term on the left hand side is equal to 1 at $r = r(\mu)$. By A3., $\frac{f_a(r|\alpha(\mu))}{f(r|\alpha(\mu))}$ is strictly increasing.

Because $\lim_{\lambda \rightarrow 0} \lim_{v \rightarrow 0} \hat{\mu}_2(r, \mu) = 1$ I can choose $\bar{v}_2 > \bar{\lambda}_2 > 0$ such that that the last expression is finite for $\lambda < \bar{\lambda}_2$ and $v \leq \bar{v}_2$ and any $r(\mu) > \alpha(\mu)$. Therefore,

$$\lim_{r(\mu) \rightarrow \infty} \frac{f(r(\mu)|\alpha(\mu))}{f_a(r(\mu)|\alpha(\mu))} \int_{r(\mu)}^{\infty} \hat{\mu}_2(r, \mu) f_a(r|\alpha(\mu)) dr = 0.$$

As argued in the text preceding the Lemma, since Q is bounded below, I may choose $r(\mu)$ as large as I like while still supporting positive effort. In particular, I may choose $r(\mu)$, and thus $\alpha(\mu)$, so that the following inequality holds for all $\mu > \mu_0$:

$$\delta \int_{r(\mu)}^{\infty} \hat{\mu}_2(r, \mu) f(r|\alpha(\mu)) dr - \frac{f(r(\mu)|\alpha(\mu))}{f_a(r(\mu)|\alpha(\mu))} \delta \int_{r(\mu)}^{\infty} \hat{\mu}_2(r, \mu) f_a(r|\alpha(\mu)) dr < .9$$

Therefore, $\delta \int_{r(\mu)}^{\infty} \left[Q'(\hat{\mu}(r, \mu)) f(r|\alpha(\mu)) - Q'(\hat{\mu}(r, \mu)) \frac{f(r(\mu)|\alpha(\mu))}{f_a(r(\mu)|\alpha(\mu))} f_a(r|\alpha(\mu)) \right] \hat{\mu}_2(r, \mu) dr < .9K$.

The maximum value Q can take is the value of exerting minimum effort (0) and holding office forever: $\frac{u(0)}{1-\delta}$. Thus, because

$$\left| \int_{r(\mu)}^{\infty} f_{aa}(r|\alpha(\mu)) dr \right| < B \text{ for some } B > 0 \text{ I can conclude that}$$

$\left| \int_{r(\mu)}^{\infty} \alpha'(\mu) Q(\hat{\mu}(r, \mu)) f_{aa}(r|\alpha(\mu)) dr \right| < B \frac{v}{\mu_0^2} \frac{u(0)}{1-\delta}$ where v is determined by the choice of $r(\mu)$ made above.

Similarly, by assumption $|u''(\alpha(\mu))| < \infty$. I may focus on a closed interval $a \in [0, \frac{v}{\mu_0}]$ so that the second derivative is uniformly bounded above:

$$|u''(\alpha(\mu))| < U \text{ for some } U > 0.$$

Using these bounds, I have that the absolute value of the derivative above is bounded by:

$$0.9K + \frac{f(r(\mu)|\alpha(\mu))}{f_a(r(\mu)|\alpha(\mu))} \frac{v}{\mu_0^2} \left(U + B \frac{u(0)}{1-\delta} \right) < K$$

The first term is strictly less than K . The second term does not depend on K , so that choosing K high enough makes it strictly less than $0.1K$. ■

From this point forward, I use the following notation:

$$\bar{\lambda} \equiv \min\{\bar{\lambda}_1, \bar{\lambda}_2\} \text{ and } \bar{v} \equiv \min\{\bar{v}_1, \bar{v}_2\} \quad (8.8)$$

Having a bounded derivative also ensures that the class of functions $T^v(\hat{C})$ is equicontinuous. A class of functions is equicontinuous if, given $\varepsilon > 0$, there is a $\delta > 0$ such that $|f(x) - f(y)| < \varepsilon$ whenever $|x - y| < \delta$ for any x in the domain of f and any $f \in T^v(\hat{C})$.

Lemma 2. *Suppose $\lambda < \bar{\lambda}$ and $v \leq \bar{v}$, and let $T^v(\hat{C})$ be a class of bounded, continuous and differentiable functions with a uniformly bounded derivative. Then $T^v(\hat{C})$ is equicontinuous.*

Proof. For any $f \in T^v(\hat{C})$, $\frac{|f(x) - f(y)|}{|x - y|} \approx f'(x)$. Because $|f'(x)| < B$, $|f(x) - f(y)| < B|x - y|$.

Because the bound B on the derivative is the same for all $f \in T^v(\hat{C})$, if we choose x and y such that $|x - y| < \frac{\varepsilon}{B}$, $|f(x) - f(y)| < \varepsilon$ for any $f \in T^v(\hat{C})$. Therefore $T^v(\hat{C})$ is equicontinuous. ■

Next I verify that the operator T^v is continuous.

Lemma 3. *The operator T^v is continuous.*

Proof. Let $\{Q_i\}_{i \in \mathbb{N}} \subset \hat{C}$ be a sequence of functions converging to Q in the sup norm.

Then, for any $\beta > 0 \exists j \in \mathbb{N}$ such that $\forall i > j, \|Q_i - Q\| < \beta$.

$$(T^v(Q_i) - T^v(Q))(\mu) = \delta \int_{r_Q(\mu)}^{\infty} [Q_i(\hat{\mu}(r, \mu)) - Q(\hat{\mu}(r, \mu))] f(r|\alpha(\mu)) dr \\ + \delta \int_{r_{Q_i}(\mu)}^{r_Q(\mu)} Q_i(\hat{\mu}(r, \mu)) f(r|\alpha(\mu)) dr$$

if $r_{Q_i}(\mu) > r_Q(\mu)$. For the reverse case, an identical argument may be used.

The first term converges to zero by definition of Q_i .

The second term converges to zero because $r_{Q_i}(\mu) \rightarrow r_Q(\mu)$. To see this, consider the following equality derived from the politician's FOC:

$$\int_{r_Q(\mu)}^{\infty} [Q_i(\hat{\mu}(r, \mu)) - Q(\hat{\mu}(r, \mu))] f_a(r|\alpha(\mu)) dr = \int_{r_{Q_i}(\mu)}^{r_Q(\mu)} Q_i(\hat{\mu}(r, \mu)) f_a(r|\alpha(\mu)) dr$$

Again, the term on the LHS converges to zero by convergence of Q_i . Hence the RHS must also converge to zero. However, because $r_{Q_i}(\mu) > \alpha(\mu)$ and $Q_i(\hat{\mu}(r, \mu)) \geq u(0)$, the terms inside the integral are bounded away from zero. Therefore, it must be that $r_{Q_i}(\mu) \rightarrow r_Q(\mu)$.

I have now established that $\|T^v(Q_i) - T^v(Q)\| \rightarrow 0$ so that T^v is a continuous operator. ■

I may now apply Schauder's FPT to find a value function and a reputation-dependent cutoff function $r(\mu)$ implementing effort strategy $\alpha(\mu, v)$ for parameter values $\lambda < \bar{\lambda}$ and $v \in [\lambda, \bar{v}]$.

This completes the proof of Theorem 2.

8.2. Impossibility of Markov perfect equilibria with positive effort - proof of Proposition 1

In this section I present a proof of a slightly more general version of Proposition 1. Specifically, I generalize the statement to include strategies which are Markovian with probability 1.

Definition 13. *An equilibrium is essentially Markov perfect if strategies (σ, α) are measurable with respect to the Markovian partition for a set of reputations $M \subset [0, 1]$ of Lebesgue measure 1.*

Note that any Markovian strategy is also essentially Markovian. Although the distinction is not of interest in and of itself, I make it here as it is useful in establishing Proposition 3 in Section 5. The extension does not significantly complicate the proof since it requires only that we note that non-Markovian strategies which are played with probability 0 do not affect the strategic calculus of players involved.

Proposition 5. *There is no essentially Markov perfect equilibrium with $V(\mu_0) > \lambda$.*

In what follows, for ease of exposition I write $\hat{Q}(\hat{\mu}(r, \mu))$ for the effective value to an incumbent of having a reputation $\hat{\mu}(r, \mu)$: $\sigma(\hat{\mu}(r, \mu))Q(\hat{\mu}(r, \mu))$.

The proof proceeds as follows. First, I consider the case in which effort is bounded below for some interval $[m, 1]$ of reputations and \hat{Q} is weakly monotonic. This leads me to conclude that \hat{Q} is unbounded, a contradiction.

Then, I generalize the result in several ways. First, if \hat{Q} is not weakly monotonic, I show that one may look at a moving average of \hat{Q} and that repeated application of the moving average operator leads to a function which is monotonic or approximately constant over an interval $[z, 1)$, and thus to the same contradiction as above.

Once this is done, I am left with the possibility that effort is not bounded below. However, I show that, if $V(\mu_0) > \lambda$ and λ is small enough, politicians with high reputation must be reelected with positive probability and, if that is the case, there must be politicians of arbitrarily high reputation who exert effort above some fixed lower bound. I complete the argument by showing that these conditions lead to the conclusion that \hat{Q} is unbounded. Thus, there can be no Markov perfect equilibrium supporting $V(\mu_0) > \lambda$ if the politician's payoffs are bounded.

Step 1: effort bounded below and \hat{Q} monotonic

Consider first the case in which there is a lower bound $b > 0$ on the effort exerted by politicians with reputation in $[x, 1)$. Using the politician's FOC, I know that his value function must satisfy

$$\delta \int_{-\infty}^{\infty} \hat{Q}(\hat{\mu}(r, \mu)) f_a(r|\alpha(\mu)) dr \geq -u'(b) = B > 0 \quad (8.9)$$

By 8.1 I can rewrite $\hat{Q}(\hat{\mu}(r, \mu)) f_a(r|\alpha(\mu))$ as $\hat{Q}(\hat{\mu}(r + \alpha(\mu), \mu)) f_a(r|0)$.

Because $\int_0^{\infty} f_a(r|0) dr < \infty$, I can find a value $r^* \in \mathbb{R}_+$ such that

$$\delta \int_{-r^*}^{r^*} \hat{Q}(\hat{\mu}(r + \alpha(\mu), \mu)) f_a(r|0) dr \geq \delta \int_{-\infty}^{\infty} \hat{Q}(\hat{\mu}(r + \alpha(\mu), \mu)) f_a(r|0) dr - \frac{B}{2}$$

Suppose \hat{Q} is weakly monotonic. If \hat{Q} is weakly decreasing, the integrals above will be weakly negative since, by symmetry (A4.), $\delta \int_{-\infty}^{\infty} \hat{Q}(\hat{\mu}(r + \alpha(\mu), \mu)) f_a(r|0) dr = \delta \int_0^{\infty} \left[\hat{Q}(\hat{\mu}(r + \alpha(\mu), \mu)) - \hat{Q}(\hat{\mu}(-r + \alpha(\mu), \mu)) \right] f_a(r|0) dr < 0$. Thus, the FOC will not be satisfied.

Suppose \hat{Q} is weakly increasing. By the Monotone Likelihood Ratio Property (A3.), I know that there is a unique point at which $f_a(r|0) = 0$ with the derivative being negative to the left and positive to the right of that point. Because $f(r|0)$

is Symmetric (A4.), this point is 0. Then,

$$\begin{aligned} & \delta \int_{-r^*}^{r^*} \hat{Q}(\hat{\mu}(r + \alpha(\mu), \mu)) f_a(r|0) dr \\ & \leq \delta \int_0^{r^*} \hat{Q}(\hat{\mu}(r^* + \alpha(\mu), \mu)) f_a(r|0) dr + \delta \int_{-r^*}^0 \hat{Q}(\hat{\mu}(-r^* + \alpha(\mu), \mu)) f_a(r|0) dr \\ & \leq \delta \left[\hat{Q}(\hat{\mu}(r^* + \alpha(\mu), \mu)) - \hat{Q}(\hat{\mu}(-r^* + \alpha(\mu), \mu)) \right] k \end{aligned}$$

where $k = \int_0^{r^*} f_a(r|0) dr$.

Therefore, $\hat{Q}(\hat{\mu}(r^* + \alpha(\mu), \mu)) - \hat{Q}(\hat{\mu}(-r^* + \alpha(\mu), \mu)) \geq \frac{B}{2\delta k} > 0$ for all μ .

Given μ and r^* , there is a μ' such that $\mu = \hat{\mu}(-r^* + \alpha(\mu'), \mu')$. Therefore, \hat{Q} must increase by at least $\frac{B}{2\delta k}$ over $[\hat{\mu}(-r^* + \alpha(\mu'), \mu'), \hat{\mu}(r^* + \alpha(\mu'), \mu')]$. Because this process can be repeated indefinitely, this implies that \hat{Q} grows without bound, which is a contradiction. Therefore, there can be no Markov perfect equilibrium in which \hat{Q} is weakly monotonic over any interval $[x, 1]$ while effort is bounded below by $b > 0$.

Step 2: non-monotonic \hat{Q}

I am left with the possibility of a \hat{Q} which is non-monotonic over every interval of the form $[x, 1]$. Suppose I have found such a \hat{Q} . Then,

$$\delta \int_{-r^*}^{r^*} \hat{Q}(\hat{\mu}(r + \alpha(\mu), \mu)) f_a(r|0) dr \geq \frac{B}{2} \text{ for all } \mu.$$

Define $\hat{Q}(x) = \hat{Q}(\hat{\mu}(r + \alpha(x), x))$ for $x \in [m, 1]$. Then,

$$\delta \int_{-r^*}^{r^*} \hat{Q}(x) f_a(r|0) dr \geq \frac{B}{2}$$

Therefore, $\delta \int_{-r^*}^{r^*} \frac{1}{\hat{\mu}(r^*, \mu) - \mu} \int_{\mu}^{\hat{\mu}(r^*, \mu)} \hat{Q}(x) dx f_a(r|0) dr \geq \frac{B}{2}$

$\frac{1}{\hat{\mu}(r^*, \mu) - \mu} \int_{\mu}^{\hat{\mu}(r^*, \mu)} \hat{Q}(x) dx$ is a moving average of \hat{Q} which can be applied repeatedly.

Definition 14. The moving average operator \hat{Q}_i , $i \in \mathbb{N} \cup 0$, is defined as

- $\hat{Q}_0 = \hat{Q}$ and
- $\hat{Q}_i(\mu) = \frac{1}{\hat{\mu}(r^*, \mu) - \mu} \int_{\mu}^{\hat{\mu}(r^*, \mu)} \hat{Q}_{i-1}(x) dx$.

Definition 15. For $\eta \geq 0$, a function f is η -approximately constant if there exists a constant function C such that $\|f - C\|_{\infty} < \eta$.

The following lemma establishes a basic but useful fact about the moving average operator.

Lemma 4. *Given a function \hat{Q} , there exists an interval of positive length $[z, 1)$ such that \hat{Q}_2 is either weakly monotonic or approximately constant on $[z, 1)$.*

Proof. After the moving average operator has been applied once, \hat{Q}_1 is continuous and differentiable with derivative

$$\hat{Q}'_1(\mu) = \frac{\partial(\frac{1}{\hat{\mu}(r^*, \mu) - \mu})}{\partial \mu} \int_{\mu}^{\hat{\mu}(r^*, \mu)} \hat{Q}_0(x) dx + \frac{1}{\hat{\mu}(r^*, \mu) - \mu} \left(\hat{\mu}_2(r^*, \mu) \hat{Q}_0(\hat{\mu}(r^*, \mu)) - \hat{Q}_0(\mu) \right).$$

Therefore \hat{Q}_2 is continuously differentiable and

$$\begin{aligned} \hat{Q}'_2 &= \frac{\partial(\frac{1}{\hat{\mu}(r^*, \mu) - \mu})}{\partial \mu} \int_{\mu}^{\hat{\mu}(r^*, \mu)} \hat{Q}_1(x) dx + \frac{1}{\hat{\mu}(r^*, \mu) - \mu} \left(\hat{\mu}_2(r^*, \mu) \hat{Q}_1(\hat{\mu}(r^*, \mu)) - \hat{Q}_1(\mu) \right). \\ \hat{Q}''_2 &= \frac{\partial^2(\frac{1}{\hat{\mu}(r^*, \mu) - \mu})}{\partial \mu^2} \int_{\mu}^{\hat{\mu}(r^*, \mu)} \hat{Q}_1(x) dx + 2 \frac{\partial(\frac{1}{\hat{\mu}(r^*, \mu) - \mu})}{\partial \mu} \left(\hat{\mu}_2(r^*, \mu) \hat{Q}_1(\hat{\mu}(r^*, \mu)) - \hat{Q}_1(\mu) \right) \\ &\quad + \frac{1}{\hat{\mu}(r^*, \mu) - \mu} \left(\hat{\mu}_{22}(r^*, \mu) \hat{Q}_1(\hat{\mu}(r^*, \mu)) + \hat{\mu}_2(r^*, \mu) \hat{Q}'_1(\hat{\mu}(r^*, \mu)) - \hat{Q}'_1(\mu) \right). \end{aligned}$$

Because \hat{Q} is bounded, \hat{Q}'_2 and \hat{Q}''_2 are bounded. Let $B > 0$ denote the bound on \hat{Q}''_2 .

Given an $\varepsilon > 0$, there is a z such that if $|\hat{Q}'_2(\mu)| > \varepsilon$ for some $\mu \in [z, 1)$ then \hat{Q}_2 is strictly monotonic over $[z, 1)$. This is because the most \hat{Q}'_2 can change in a distance less than $1 - z$ is $B(1 - z) < \varepsilon$ for z close enough to 1. If there is no $\mu \in [z, 1)$ such that $|\hat{Q}'_2(\mu)| > \varepsilon$, then $\|\hat{Q}_2 - C\|_{\infty} < \eta$ for some constant function C and a η which becomes arbitrarily small as $\varepsilon \rightarrow 0$. Thus, for any $\eta > 0$, we can choose z so that \hat{Q}_2 is η -approximately constant over $[z, 1)$. ■

If \hat{Q}_2 is weakly monotonic over $[z, 1)$, I may now repeat the arguments for weakly monotonic functions on \hat{Q}_2 starting at the point z . Since a bounded \hat{Q} should imply a bounded \hat{Q}_2 , I am once again left with a contradiction.

If \hat{Q}_2 is merely approximately constant, I note that $\delta \int_{-r^*}^{r^*} C f_a(r|0) dr = 0$ by Symmetry of $f(r|0)$ (A4.) and, for μ such that $\hat{\mu}(-r^* + \alpha(\mu), \mu) > z$,

$$\begin{aligned} &\left| \int_{-r^*}^{r^*} \hat{Q}_2(\hat{\mu}(r + \alpha(\mu), \mu)) f_a(r|0) dr - \int_{-r^*}^{r^*} C f_a(r|0) dr \right| \\ &= \left| \int_{-r^*}^{r^*} \hat{Q}_2(\hat{\mu}(r + \alpha(\mu), \mu)) f_a(r|0) dr \right| < \frac{B}{2} \text{ (if } \eta \text{ is chosen small enough) which} \end{aligned}$$

contradicts the derived properties of \hat{Q} .

Step 3: no lower bound on effort

Now, I consider the case where there is no lower bound on effort exerted. The following Lemmas provide constraints on what can happen in such a hypothetical equilibrium.

Lemma 5. *There exists a $\bar{\lambda}_M > 0$ such that, when $\lambda < \bar{\lambda}_M$, in any Markov equilibrium with $V(\mu_0) > \lambda$, every reputation interval $[\mu, 1]$ must contain reputation points at which politicians are reelected with strictly positive probability.*

Proof. Suppose not. Let $\hat{r}(a)$ denote the outcome which would leave the incumbent's reputation unchanged:

$$\hat{r}(a) = \{r | \hat{\mu}(r, \mu) = \mu\}$$

Note that, using MLRP (A3.), $\hat{r}(a) < a$ (if r is normally distributed $\hat{r}(a) = \frac{a}{2}$).

Let $\varepsilon > 0$ and $\bar{\mu}(\varepsilon) = \sup\{\mu \in [0, 1] \text{ s.t. } \sigma(\mu) \geq \varepsilon\}$. For small enough ε , the FOC of a politician with reputation $\bar{\mu}(\varepsilon)$ is:

$$u'(\alpha(\bar{\mu}(\varepsilon))) + \delta \int_{-\infty}^{\hat{r}(a)} Q(\hat{\mu}(r, \bar{\mu}(\varepsilon))) f_a(r | \alpha(\bar{\mu}(\varepsilon))) dr + O(\varepsilon) < 0 \text{ for any } \alpha(\bar{\mu})$$

where $O(\varepsilon)$ approaches zero as ε does. The expression is negative because $f_a(r | \alpha(\bar{\mu}(\varepsilon)))$ is negative for all values below $\alpha(\bar{\mu}(\varepsilon))$. Clearly, the FOC for a politicians with reputation greater than $\bar{\mu}(\varepsilon)$ will also be negative. Furthermore, because the politician's FOC is strictly negative, we can find an $m > 0$, such that:

$$u'(\alpha(\bar{\mu}(\varepsilon) - m)) + \delta \int_{-\infty}^{\hat{r}(a)} Q(\hat{\mu}(r, \bar{\mu}(\varepsilon) - m)) f_a(r | \alpha(\bar{\mu}(\varepsilon) - m)) dr + O(\varepsilon) < 0 \text{ for any } \alpha(\bar{\mu})$$

Therefore, $\alpha(\mu) = 0$ for all $\mu \in (\bar{\mu}(\varepsilon) - m, 1)$.

Note that V is bounded above by the constant $\bar{V} = \frac{\lambda + \bar{a}}{1 - \delta}$ where $u(\bar{a}) = 0$. Let k satisfy $\sum_{i=0}^k \delta^i \lambda + \delta^{k+1} \bar{V} < V(\mu_0)$.

As λ approaches zero, updating when $a = 0$ becomes arbitrarily slow. Thus, for any given probability p , we can find a $\bar{\lambda}_M > 0$ which ensures that, if $\lambda < \bar{\lambda}_M$, the incumbent's reputation will leave the interval $(\bar{\mu}(\varepsilon) - m, 1)$ in fewer than k periods with probability no greater than p . Choosing p small enough, this yields an upper bound on the value to the voter of having a politician with reputation $\bar{\mu}(\varepsilon)$ in office which is lower than the value of electing a challenger:

$$V(\bar{\mu}(\varepsilon)) < (1 - p) \left(\sum_{i=0}^k \delta^i \lambda + \delta^{k+1} \bar{V} \right) + p \delta \bar{V} < V(\mu_0)$$

If the politician is reelected in each of his first k terms. If he does not survive k terms, then $V(\hat{\mu})$ is less than:

$$V(\hat{\mu}) < \lambda + \delta V(\mu_0) < V(\mu_0)$$

I conclude that it is not a best response for the voter to reelect a politician with reputation $\bar{\mu}(\varepsilon)$, contradicting the definition of $\bar{\mu}(\varepsilon)$. ■

Lemma 6. *Let $\lambda < \bar{\lambda}_M$ and consider a Markov perfect equilibrium with $V(\mu_0) > \lambda$. In every reputation interval $[\mu, 1]$ there must be a subset of positive measure in which politicians exert effort above some fixed lower bound $b > 0$.*

Proof. Suppose not. Then, choose a lower bound $b < \frac{1}{2}V(\mu_0)$ and let $[\mu, 1]$ be an interval over which effort is bounded above by b almost everywhere. V is bounded above by the constant $\bar{V} = \frac{\lambda + \bar{a}}{1 - \delta}$ where $u(\bar{a}) = 0$. Let k satisfy $\sum_{i=0}^k \delta^i b + \delta^{k+1} \bar{V} < V(\mu_0)$. By Lemma 5, there must be reputations arbitrarily close to 1 which are reelected with positive probability. Because effort is bounded, I may choose a reputation (call it m) which is reelected with positive probability and from which the probability of transitioning out of $[\mu, 1]$ in k periods or fewer (call it p) is arbitrarily small. In particular, if I choose $p < \frac{V(\mu_0) - (\sum_{i=0}^k \delta^i \lambda + \delta^{k+1} \bar{V})}{\delta \bar{V} - (\sum_{i=0}^k \delta^i \lambda + \delta^{k+1} \bar{V})}$, an upper bound on the value to the voter of having a politician with reputation m in office

$(V(m))$ is:

$$V(m) < (1 - p) \left(\sum_{i=0}^k \delta^i b + \delta^{k+1} \bar{V} \right) + p \delta \bar{V} < V(\mu_0)$$

If the politician is reelected in each of his first k terms. Note that the probability of transitioning to a point in $[\mu, 1]$ at which effort higher than b is exerted is zero because this may happen only on a subset of measure 0, and therefore this possibility does not affect the calculation of expected rewards.

If he does not survive k terms, then $V(m)$ is less than:

$$V(m) < b + \delta V(\mu_0) < V(\mu_0)$$

Therefore, it is not a best response to reelect a politician when his reputation is m , which contradicts the definition of m . ■

Given Lemma 6, if I have a weakly monotonic value function I need only to modify the arguments above as follows. Instead of moving to a reputation satisfying $\mu = \hat{\mu}(-r^* + \alpha(\mu'), \mu')$ I move to one satisfying $\alpha(\mu') > b$ and $\mu < \hat{\mu}(-r^* + \alpha(\mu'), \mu')$. Once again, I conclude that \hat{Q} must increase by at least a fixed amount $\frac{B}{2\delta k}$ infinitely many times, contradicting its boundedness.

To deal with non-monotonic candidate value functions \hat{Q} I note that, given Lemma 6, repeated application of the moving average operation ensures that the value of all integrals $\delta \int_{-r^*}^{r^*} \hat{Q}_i(\hat{\mu}(r, \mu)) f_a(r|\alpha(\mu)) dr$ will be positive. Because these are defined on a closed set $[\mu', 1]$, there exists a minimum value of these integrals. Now, I may apply the same arguments as above: \hat{Q}_2 includes a weakly monotonic segment $[z, 1)$, and this contradicts the boundedness of \hat{Q} .

Finally, note that in all the arguments above, having a function Z which differs from \hat{Q} only on a set of Lebesgue measure 0 will not change any of the results, because the integrals will yield the same values under both functions. Therefore, it is immediate that the result extends to rule out essentially Markov perfect equilibria with positive value for the voter.

References

- [1] Abreu, D., D. Pearce, and E. Stachetti. 1990. "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring." *Econometrica* 58(5): 1041-63.
- [2] Ashworth, S. 2005. "Reputational Dynamics and Political Careers." *Journal of Law, Economics, & Organization* 21(2): 441-466.
- [3] Ashworth, S. and E. Bueno de Mesquita. 2006. "Delivering the Goods: Legislative Particularism in Different Electoral and Institutional Settings" *The Journal of Politics* 68: 168-179.
- [4] Ashworth, S. and E. Bueno de Mesquita. 2008. "Electoral Selection, Strategic Challenger Entry, and the Incumbency Advantage." *The Journal of Politics* 70: 1006-1025.
- [5] Ashworth, S., E. Bueno de Mesquita and A. Friedenbergl. 2010. "Creating Incentives and Selecting Good Types Revisited." Unpublished manuscript.
- [6] Bagnoli, M. and T. Bergstrom. 2005. "Log-concave Probability and its Applications." *Economic Theory* 26: 445-469.
- [7] Banks, J. and J. Duggan. 2008. "A Dynamic Model of Democratic Elections in Multidimensional Policy Spaces." *Quarterly Journal of Political Science* 3: 269-299.
- [8] Banks, J. and R. Sundaram. 1993. "Adverse Selection and Moral Hazard in a Repeated Elections Model." In *Political Economy: Institutions, Information, Competition, and Representation*, eds. W. Barnett, M.J. Hinich and N. Schofield. New York, Cambridge University Press.
- [9] Banks, J. and R. Sundaram 1998. "Optimal Retention in Agency Problems." *Journal of Economic Theory* 82: 293-323.

- [10] Barro, R. 1973. "The Control of Politicians: An Economic Model." *Public Choice* 14: 19-42.
- [11] Berganza, J. C. 2000. "Two Roles for Elections: Disciplining the Incumbent and Selecting a Competent Candidate." *Public Choice* 105: 165-193.
- [12] Besley, T. 2006. *Principled Agents? The Political Economy of Good Government*. New York: Oxford University Press.
- [13] Bobonis, G., L.R. Cámara Fuertes and R. Schwabe. 2011. "The Dynamic Effects of Information on Political Corruption: Theory and Evidence from Puerto Rico." Unpublished manuscript.
- [14] Cain, B., J. Ferejohn, and M. Fiorina 1990. *The Personal Vote*. Cambridge MA, Harvard University Press.
- [15] Coate, S. and S. Morris 1995. "On the Form of Transfers to Special Interests" *Journal of Political Economy* 103(6): 1210-1235.
- [16] Dominguez-Martinez, S., O. Swank, and B. Visser "In Defense of Boards" *Journal of Economics & Management Strategy* 17(3): 667-682.
- [17] Duggan, J. 2000. "Repeated Elections with Asymmetric Information." *Economics and Politics* 12(2): 109-135.
- [18] Farrell, J. and E. Maskin 1989. "Renegotiation in Repeated Games." *Games and Economic Behavior* 1: 327-360.
- [19] Fearon, J. 1999. "Electoral Accountability and the Control of Politicians: Selecting Good Candidates versus Sanctioning Poor Performance." In *Democracy, Accountability, and Representation*, eds. A. Przeworski, S.C. Stokes and B. Manin. New York, Cambridge University Press.
- [20] Ferejohn, J. 1986. "Incumbent Performance and Electoral Control." *Public Choice* 50: 5-25.

- [21] Galasso, V., M. Landi, A. Merlo and A. Matozzi 2009. "The Labor Market of Italian Politicians." Unpublished manuscript.
- [22] Holmström, B. 1999. "Managerial Incentive Problems: A Dynamic Perspective." *Review of Economic Studies* 66: 169-182.
- [23] Hörner, J. 2002. "Reputation and Competition." *American Economic Review* 92: 644-663.
- [24] Key, V. O., Jr. 1966. *The Responsible Electorate*. Cambridge MA, Harvard University Press.
- [25] Levitt, S.D. 1996. "How Do Senators Vote? Disentangling the Role of Voter Preferences, Party Affiliation, and Senator Ideology" *American Economic Review* 86(3): 425-41.
- [26] Lohmann, S. 1998. "An Information Rationale for the Power of Special Interests" *American Political Science Review* 92(4): 809-827.
- [27] Lusternik, L.A. and V.I. Sobolev 1974. *Elements of Functional Analysis*. New York, John Wiley and Sons.
- [28] Mailath, G.J. and L. Samuelson 2001. "Who Wants a Good Reputation?" *Review of Economic Studies* 68: 415-441.
- [29] Maskin, E. and J. Tirole 2001. "Markov Perfect Equilibrium: I. Observable Actions." *Journal of Economic Theory* 100: 191-219.
- [30] Meirowitz, A. 2007. "Probabilistic Voting and Accountability in Elections with Uncertain Policy Constraints." *Journal of Public Economic Theory* 9(1): 41-68.
- [31] Milgrom, P.R. 1981. "Good News and Bad News: Representation Theorems and Applications." *Bell Journal of Economics* 12(2): 380-391.

- [32] Pearce, D. 1987. "Renegotiation-Proof Equilibria: Collective Rationality and Intertemporal Cooperation." Cowles Foundation Discussion Paper No. 855.
- [33] Persson, T. and G. Tabellini 2000. *Political Economics: Explaining Economic Policy*. Cambridge MA, MIT Press.
- [34] Reed, R.W. 1994. "A Retrospective Voting Model With Heterogeneous Politicians." *Economics and Politics* 6(1): 39-58.
- [35] Svulik, M. 2010. "Learning to Love Democracy: Electoral Accountability, Government Performance, and the Consolidation of Democracy." Unpublished manuscript.
- [36] Smart, M. and D.M. Sturm. 2006. "Term Limits and Electoral Accountability." CEPR Discussion Paper no. 4272.
- [37] Zaller, J. 1998. "Politicians as Prize Fighters: Electoral Selection and the Incumbency Advantage." In *Politicians and Party Politics*, ed. J.G. Geer. Baltimore, MD, Johns Hopkins University Press.